

<https://doi.org/10.34024/prometeica.2026.33.21454>

THE THEORY OF MENTAL MODELS WITHIN A NON-AXIOMATIC LOGIC

LA TEORÍA DE LOS MODELOS MENTALES DENTRO DE UNA LÓGICA NO AXIOMÁTICA

A TEORIA DOS MODELOS MENTAIS DENTRO DE UMA LÓGICA NÃO AXIOMÁTICA

Miguel López-Astorga

(Universidad de Talca, Chile)

milopez@utalca.cl

Recibido: 14/11/2025

Aprobado: 29/04/2026

ABSTRACT

This paper tries to capture the theory of mental models from Non-Axiomatic Logic. The theory of mental models describes people's natural inferential processes and has strong empirical support. Non-Axiomatic Logic allows building a computer system based on itself. So, if the theory of mental models is right, and Non-Axiomatic Logic can capture the theory, it is possible to build a computer system based on the latter logic coming to conclusions like those of the human natural inferential ability. I attempt to show that with several components of Non-Axiomatic Logic, this logic can represent the basic inferential processes the theory of mental models proposes.

Keywords: computer system. human reasoning. inference. non-axiomatic logic. theory of mental models.

RESUMEN

Este trabajo intenta abordar la teoría de los modelos mentales desde Non-Axiomatic Logic (Lógica No Axiomática). La teoría de los modelos mentales describe procesos inferenciales naturales de las personas y tiene un fuerte apoyo empírico. Non-Axiomatic Logic permite construir un sistema computacional basado en ella. Por tanto, si la teoría de los modelos mentales es correcta y Non-Axiomatic Logic puede representar la teoría, es posible elaborar un sistema computacional basado en esta última lógica que llegue a conclusiones como las de la capacidad inferencial natural humana. Intento mostrar que, con varios elementos de Non-Axiomatic Logic, esta lógica puede recoger procesos inferenciales básicos que la teoría de los modelos mentales propone.

Palabras clave: sistema computacional. razonamiento humano. inferencia. non-axiomatic logic. teoría de los modelos mentales.

RESUMO

Este trabalho procura abordar a teoria dos modelos mentais a partir da Non-Axiomatic Logic (Lógica Não Axiomática). A teoria dos modelos mentais descreve os processos inferenciais naturais das pessoas e tem um forte suporte empírico. A Non-Axiomatic Logic permite construir um sistema computacional baseado nela. Portanto, se a teoria dos modelos mentais está correta e a Non-Axiomatic Logic pode representar a teoria, é possível desenvolver um sistema computacional baseado nesta última lógica que chegue a conclusões semelhantes à capacidade inferencial natural humana. Procuro demonstrar que, com vários elementos da Non-Axiomatic Logic, esta lógica pode representar processos inferenciais básicos propostos pela teoria dos modelos mentais.

Palavras-chave: sistema computacional. raciocínio humano. inferência. non-axiomatic logic. teoria dos modelos mentais.

Introduction¹

We can ask a question: will we be able to build a computer program thinking like us? The answer is complex. We could build that program from a psychological theory describing the way human beings think. Do we have such a theory? Perhaps we do. At least, we have a psychological theory with strong empirical evidence: the theory of mental models (e.g., Johnson-Laird, Byrne, & Khemlani, 2024).

In this paper, I will assume that the theory of mental models (from now on, TMM) is correct, and its descriptions of the manner people make inferences are suitable. My aim is to show that we can create a computer program coming to the same conclusions as TMM. I will argue that if TMM is right, it is possible to build a computer system thinking as human beings.

Specifically, I will attempt to show that the general principles and theses of TMM can be captured from Non-Axiomatic Logic (Wang, 2013). Non-Axiomatic Logic, or NAL, is the logic underlying a computer system such as Non-Axiomatic Reasoning System (e.g., Wang, 2006), or NARS. If NAL can accept and reject the same conclusions as TMM, NARS can do that, too. But if a computer system such as NARS can come to the same conclusions as those TMM predicts, it is possible to argue that NARS can work as the human mind.

The first section of this paper will explain the general principles and theses of TMM. After that, I will describe the components of NAL necessary to translate those principles and theses into ‘Narsese’, that is, the language to communicate with NARS (e.g., Wang, 2013). Finally, I will try to show that NAL can come to the same conclusions as TMM.

TMM, conjunctions of possibilities, and necessity

If a concept is important within TMM (at least, within its latest version), it is that of ‘conjunction of possibilities’ (e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017). The theory assigns conjunctions of possibilities (i.e., sets of possibilities linked by copula ‘and’) to classical logic connectives such as the conditional (i.e., copula ‘if...then’), biconditional (i.e., copula ‘if and only if’), disjunction (i.e., copula ‘or’, and conjunction (i.e., copula ‘and’) (see also, e.g., Khemlani, Byrne, & Johnson-Laird, 2018). Resorting to the language of modal logic just as a metalanguage, we can express the conjunction of possibilities of each connective as follows.

¹ **Acknowledgments:** Project “Dialéctica virtuosa de la educación. De la fundamentación filosófica y antropológica a implementaciones concretas.” Project code: 15/I139-SAL160/23. UNMdP.

-A conditional such as ‘if I have milk, then I have lactose’: $\Diamond(Milk \wedge Lactose) \wedge \Diamond(\neg Milk \wedge Lactose) \wedge \Diamond(\neg Milk \wedge \neg Lactose)$ (see also, e.g., Quelhas, Rasga, & Johnson-Laird, 2017).

Where ‘ $\Diamond$ ’ indicates that what follows it between brackets is possible, ‘ $\wedge$ ’ stands for copula ‘and’, and ‘ $\neg$ ’ expresses that what is to its right is negated, i.e., represents negation.

The proponents of TMM usually claim that the theory is not a standard modal logic (see also, e.g., Espino, Byrne, & Johnson-Laird, 2020). I am respecting that here. As said, the language of modal logic, especially symbol ‘ $\Diamond$ ’, is being used only as a metalanguage (for a discussion about whether TMM is a modal logic, see also, e.g., Johnson-Laird & Ragni, 2025). The same applies to the cases of other connectives in this section.

If a conditional is mentally processed as a biconditional (i.e., as a proposition with copula ‘if and only if’), then the second possibility in the example above (that in which *Milk* is negated and *Lactose* is affirmed) is removed (see also, e.g., Johnson-Laird & Byrne, 2002). On the other hand, only the first conjunct (that in which both *Milk* and *Lactose* are affirmed) is intuitively obvious. The other two conjuncts need deliberative effort to be identified (see also, e.g., Barres & Johnson-Laird, 2003). Besides, TMM claims that the conjuncts do not always match the cases in which the conditional holds in classical logic. Semantics and pragmatics can change, or modulate, them (e.g., Johnson-Laird & Byrne, 2002). If the conditional is ‘if I go to China, then I go to Beijing’, the conjunction of possibilities is: $\Diamond(China \wedge Beijing) \wedge \Diamond(China \wedge \neg Beijing) \wedge \Diamond(\neg China \wedge \neg Beijing)$. The second conjunct has a different structure now. The clause that is negated is *Beijing*, that is, the second one. The reason is that I cannot travel to Beijing without going to China.

-A disjunction such as ‘I have milk or I have juice’: $\Diamond(Milk \wedge Juice) \wedge \Diamond(Milk \wedge \neg Juice) \wedge \Diamond(\neg Milk \wedge Juice)$ (see also, e.g., Johnson-Laird, Khemlani, & Goodwin, 2015).

If a disjunction is mentally processed as exclusive (i.e., as a disjunction in which the two disjuncts cannot be true at the same time), the first conjunct (that in which both *Milk* and *Juice* are the case) is eliminated (see also, e.g., Quelhas, Rasga, & Johnson-Laird, 2019). There is also a difference in disjunction between the conjunctions of possibilities grasped by intuition and the conjunctions of possibilities apprehended by deliberative effort. The previous conjunction only is considered when people make deliberative effort. If not, the conjunction of possibilities is, in the case when individuals understand the disjunction as inclusive (i.e., as a disjunction in which the two disjuncts can be true at the same time), as follows: $\Diamond(Milk \wedge Juice) \wedge \Diamond(Milk) \wedge \Diamond(Juice)$ (see also, e.g., Espino & Byrne, 2013). Semantics and pragmatics can also modulate the possibilities of a disjunction (see also, e.g., Orenes & Johnson-Laird, 2012). For example, given the disjunction ‘I go to China, or I go to Beijing’, its conjunction of possibilities would be: $\Diamond(China \wedge Beijing) \wedge \Diamond(China \wedge \neg Beijing)$. The third conjunct is missing because Beijing is the capital of China.

-A conjunction such as ‘I have milk and I have lactose’: $\Diamond(Milk \wedge Lactose)$ (see also, e.g., Khemlani, Orenes, & Johnson-Laird, 2014).

Conjunctions in natural language generally refer to facts. This is because, according to TMM, when a sentence is linked to just one possibility, it is a fact (e.g., Khemlani et al., 2017). Modulation is also possible in the case of conjunction, too (see also, e.g., Johnson-Laird, 2010). For instance, the possibilities corresponding to ‘you have milk and you feel bad’ can be equivalent to those of a conditional such as ‘if you have milk, then you will feel bad’. The conjunction of possibilities would be: $\Diamond(Milk \wedge Bad) \wedge \Diamond(\neg Milk \wedge Bad) \wedge \Diamond(\neg Milk \wedge \neg Bad)$.

There are many interesting details within the framework of TMM. However, what is essential to achieve the goal of the present paper is the way it derives conclusions. On the one hand, a piece of information can remove possibilities and to lead to a conclusion. Let us see some examples.

Let us suppose that (1) holds.

(1) If I have milk, then I have lactose

Following TMM, if (1) is the case, and the necessary deliberative effort is made, (2) is the case as well.

(2) $\Diamond(Milk \wedge Lactose) \wedge \Diamond(\neg Milk \wedge Lactose) \wedge \Diamond(\neg Milk \wedge \neg Lactose)$

The information that I drank milk eliminates the second and third conjuncts in (2). I do not have milk in them. Only one possibility of the conjunction remains: the first one. In it, not only do I have milk, but I also have lactose. For this reason, given (2) and the information that I have milk, I can conclude that I have lactose.

This is an application of Modus Ponendo Ponens, that is, the logical schema ‘if p then q, p; therefore, q’ (see also, e.g., Byrne & Johnson-Laird, 2009). The theory explains the application of, for example, Modus Tollendo Tollens, that is, the logical schema ‘if p then q, not-q; therefore, not-p, too. If (2) and the information that I did not have lactose are correct, the possibilities in which I have lactose in (2), that is, the two first ones, are removed. In the third one, I do not have milk. Therefore, I can infer the latter fact, that is, that I do not have milk either (e.g., Byrne & Johnson-Laird, 2009).

From these explanations about Modus Ponendo Ponens and Modus Tollendo Tollens, one might think that there are not any differences between TMM and classical logic. But this is not so. Beyond the fact that deliberative effort is required to use Modus Tollendo Tollens (it is necessary to identify the third conjunct in (2), i.e., the possibility in which the two clauses are negated), there are cases in which modulation prevents Modus Tollendo Tollens from being applied (e.g., Byrne & Johnson-Laird, 2009). If the conditional is (3),

(3) If I go to China, then I go to Beijing

The conjunction of possibilities will be (4).

(4) $\Diamond(China \wedge Beijing) \wedge \Diamond(China \wedge \neg Beijing) \wedge \Diamond(\neg China \wedge \neg Beijing)$

In this situation, Modus Tollendo Tollens does not allow inferring any conclusions. The information that I was not in Beijing only enables to eliminate the first possibility in (4). I was not in Beijing in two possibilities: the second and third conjuncts. In one of them, I was in China. In the other one, I was not in that country. Hence, the fact that I was not in China cannot be inferred. On the contrary, the fact that I was not in Beijing could be derived from the fact that I was not in China. The latter fact would remove the two first conjuncts in (4). In the remaining possibility, I was not in Beijing. This is against classical logic, which does not allow making that.

Similar examples can be given in the case of disjunction. If (5) is true,

(5) I have milk or I have juice

Conjunction (6) is also true (if the deliberative effort needed is made):

(6) $\Diamond(Milk \wedge Juice) \wedge \Diamond(Milk \wedge \neg Juice) \wedge \Diamond(\neg Milk \wedge Juice)$

This allows applying rules valid in classical logic such as Modus Tollendo Ponens (i.e., the rule enabling to derive a disjunct from its disjunction and the negation of the other disjunct, e.g., the rule with this structure: ‘p or q, not-p; therefore, q’) (see also, e.g., Khemlani & Johnson-Laird, 2009). If it is known that (5) is true, and I state that I do not drink milk, only the third conjunct in (6) is acceptable. In that conjunct, I drink juice.

TMM moves away from basic requirements of classical logic with its account of disjunction, too. Modulation can block the use of rules such as Modus Tollendo Ponens as well (see also, e.g., Orenes & Johnson-Laird, 2012). If the disjunction is (7),

(7) I go to China, or I go to Beijing

The possibilities will be two:

(8) $\Diamond(China \wedge Beijing) \wedge \Diamond(China \wedge \neg Beijing)$

We cannot deduce that I was in Beijing from the fact that I was not in China. In one conjunct in (8), I was in Beijing. In the other one, I was not in that city.

Lastly, we can think of an example related to conjunction. Conjunctions usually indicate facts, but there are cases in which this is different. For example, the case of (9).

(9) You have milk and you feel bad

Its conjunction of possibilities is (10).

(10) $\Diamond(Milk \wedge Bad) \wedge \Diamond(\neg Milk \wedge Bad) \wedge \Diamond(\neg Milk \wedge \neg Bad)$

As pointed out above, the possibilities in (10) are those of the conditional. So, we can apply a rule like Modus Ponendo Ponens. I can derive that I will feel bad from the fact that I drank milk. Likewise, we can use a rule akin to Modus Tollendo Tollens. I can conclude that I did not have milk from the fact that I do not feel bad.

Another important thesis of TMM is that it rejects the validity concept as understood in classical logic. According to this concept, a conclusion must be true if its premises are so. Instead, it proposes an alternative concept: necessity (e.g., Johnson-Laird et al., 2024). This concept is essential because it allows explaining interesting phenomena such as that of the deduction in (11) (see also, e.g., Orenes & Johnson-Laird, 2012).

(11) I have milk

Therefore, I have milk or I have juice

Inference (11) is correct in classical logic. If the premise ('I have milk') is true, given that the conclusion is a disjunction ('I have milk or I have juice') in which the premise is one of the disjuncts, the conclusion must be true, too. However, people tend to think that it is false (Orenes & Johnson-Laird, 2012). According to TMM, the reason is that the conclusion is not necessary. It refers to a conjunction of possibilities such as (6), that is, $\Diamond(Milk \wedge Juice) \wedge \Diamond(Milk \wedge \neg Juice) \wedge \Diamond(\neg Milk \wedge Juice)$. That makes it a non-necessary conclusion: it implies more possibilities than the premise (e.g., Johnson-Laird et al., 2024). For example, the third conjunct in (6) provides that it is possible that I do not have milk and that I have juice. That possibility is not compatible with the premise.

These are the main components of TMM (at least, as far as my aims in this paper are concerned). I will try to show that NAL has the resources to create a level or module of NARS working following those components. Given its empirical support, we can think that TMM describes the manner our mind makes inferences. If this is the case, the result of my account will be a level or module of NARS built a posteriori, and being able to come to the same conclusions as human beings.

Inheritance, extensional union, and extensional difference

A key copula in Narsese is ‘ $\rightarrow$ ’, that is, the ‘inheritance copula’ (e.g., Wang, 2023). It establishes relations between extensions and intensions in a sense different from the habitual one in logic (see also, e.g., Wang, 2013). Let us suppose that the inheritance copula links the term *Turtle* to the term *Chelonian*. This means that *Turtle* is a term included in the set of the extension of *Chelonian* (*Turtle* is an example of *Chelonian*), and that *Chelonian* is a term belonging to the set of the intension of *Turtle* (*Chelonian* is a property of *Turtle*) (e.g., Wang, 2013; Definition 2.8). This can be expressed as follows:

$$(12) \quad \textit{Turtle} \rightarrow \textit{Chelonian}$$

If $\textit{Turtle}^E$ represents the extension of *Turtle*, and $\textit{Chelonian}^E$ stands for the extension of *Chelonian*, (12) implies that

$$\textit{Turtle}^E \subseteq \textit{Chelonian}^E \text{ (Wang, 2013; Theorem 2.4).}$$

NAL resorts to set theory language as a metalanguage (e.g., Wang, 2013). Here, ‘ $\subseteq$ ’ indicates that the left set is equal or a subset of the right set.

On the other hand, if $\textit{Turtle}^I$ indicates the intension of *Turtle*, and $\textit{Chelonian}^I$ refers to the intension of *Chelonian*, (12) also provides that

$$\textit{Chelonian}^I \subseteq \textit{Turtle}^I \text{ (Wang, 2013; Theorem 2.4).}$$

NAL is a non-axiomatic logic because it can never be sure about any statement. Evidence changes (e.g., Wang, 2011). For this reason, truth values in NAL needs two numbers: it is necessary both a ‘frequency value’ (f) and a confidence value (c). If ‘ w^+ ’ represents the set of ‘positive evidence’, and ‘ w ’ is the set of ‘total evidence’, then “ $f = w^+/w$ ” (Wang, 2013, p. 35; Table 3.1). If ‘ k ’ is a constant being able to have any value, the system being less confident as k is higher (see also, e.g., Wang, 2023), then “ $c = w/(w + k)$ ” (Wang, 2013, p. 35; Table 3.1). A statement such as (12) must include the values for f and c in the same way as (13) (e.g., Wang, 2013).

$$(13) \quad \textit{Turtle} \rightarrow \textit{Chelonian} \langle f, c \rangle$$

If the system has seen 500 turtles that are chelonians, it will consider (14).

$$(14) \quad \textit{Turtle} \rightarrow \textit{Chelonian} \langle 1, 0.998 \rangle$$

Where $f = 500/500 = 1$, and $c = 500/501 = 0.998$ (for $k = 1$).

NAL also admits different kinds of compound terms. For my aims in the present paper, we only need those built from ‘extensional unions’ and ‘extensional differences’. Using First-Order Predicate Calculus as a metalanguage, which is also frequent in NAL (e.g., Wang, 2013), the extensional union, or ‘intensional intersection’, $\textit{Turtle} \cup \textit{Salamander}$ means that

$$\forall x \{ [(\textit{Turtle} \cup \textit{Salamander}) \rightarrow x] \Leftrightarrow [(\textit{Turtle} \rightarrow x) \wedge (\textit{Salamander} \rightarrow x)] \} \text{ (see, e.g., Wang, 2013; Definition 7.7).}$$

Where ‘ $\forall$ ’ is the universal quantifier, which indicates that what is in the formula applies to any x in the universe considered, and ‘ $\Leftrightarrow$ ’ is the symbol for the biconditional.

Extensional union $\textit{Turtle} \cup \textit{Salamander}$ refers to the set of all the turtles and all the salamanders, and to the properties shared by *Turtle* and *Salamander* (in their intensions). So, $(\textit{Turtle} \cup \textit{Salamander})^E$ includes both the extension of *Turtle* and the extension of *Salamander* (Wang, 2013; Theorem 7.4). Both (15) and (16) hold.

- (15) $Turtle \rightarrow (Turtle \cup Salamander) \langle 1, 0.9 \rangle$
 (16) $Salamander \rightarrow (Turtle \cup Salamander) \langle 1, 0.9 \rangle$

In (15) and (16), $f = 1$ and $c = 0.9$ because those are the numbers NAL attributes whenever f and c are not explicitly established (Wang, 2013).

Extensional difference $Turtle - Red$ indicates that

$\forall x \{[x \rightarrow (Turtle - Red)] \Leftrightarrow [(x \rightarrow Turtle) \wedge \neg(x \rightarrow Red)]\}$ (see, e.g., Wang, 2013; Definition 7.8).

Extensional difference $Turtle - Red$ represents the set of turtles that are not red. Therefore, only (17) holds. (18) cannot be accepted (Wang, 2013; Theorem 7.10).

- (17) $(Turtle - Red) \rightarrow Turtle \langle 1, 0.9 \rangle$
 (18) $(Turtle - Red) \rightarrow Red \langle 1, 0.9 \rangle$

Finally, it is only necessary to add that evaluative questions such as (19) are possible in NAL.

- (19) $Turtle \rightarrow Chelonian?$

If $Turtle \rightarrow Chelonian$ is in the system, the answer will be positive, and the values for f and c will be shown.

For several reasons that I will not consider here (to address them would take us away from my goal in the present paper), a statement can be in the system with different values. In that case, given an evaluative question on that statement, the system would indicate the highest c and the value f for that highest c (e.g., Wang, 2013). This is an important characteristic of NAL. However, as will be seen below, it is not relevant here.

NAL is much more complex than my description in this section. I have presented just a limited number of components of the system, and none of its inference rules. But that number suffices to capture the way TMM works.

TMM from NAL

What I will show below is not the first or the only possible computer program working as TMM. There are already programs trying to computationally implement TMM (e.g., Johnson-Laird, Byrne, & Khemlani, 2023; Khemlani & Johnson-Laird, 2022; Ragni, Kola, & Johnson-Laird, 2018). As far as I know, the present paper proposes the first computational treatment of TMM from NAL, and hence from NARS.

To build a level or module of NAL being able to derive conclusions like those of TMM, the first task to do is to explain how the conjunctions of possibilities can be translated into Narsese. I will try to do that in this section. I will consider each of the examples above. They are illustrative enough to describe the manner TMM understands the human inferential activity. To capture each of them can mean to capture the essential basis of the latter theory.

The first example was that of (1), that is, ‘if I have milk, then I have lactose’, and (2), that is, $\Diamond(Milk \wedge Lactose) \wedge \Diamond(\neg Milk \wedge Lactose) \wedge \Diamond(\neg Milk \wedge \neg Lactose)$. (2) can be translated as set of inheritance statements (20).

- (20) $\{(Milk \cup Lactose) \rightarrow Possible \langle 1, 0.9 \rangle, (Lactose - Milk) \rightarrow Possible \langle 1, 0.9 \rangle, (Milk - Lactose) \rightarrow Impossible \langle 1, 0.9 \rangle\}$

The first element of the set, that is, $(Milk \cup Lactose) \rightarrow Possible \langle 1, 0.9 \rangle$, stands for the possibility in which I have milk and lactose. The second one, that is, $(Lactose - Milk) \rightarrow Possible \langle 1, 0.9 \rangle$, represents the case in which I have lactose without having milk. The third one, that is, $(Milk - Lactose) \rightarrow Impossible \langle 1, 0.9 \rangle$, refers to the circumstance in which I have milk without having lactose, which is deemed as impossible.

One might think that one possibility is missing, and that one of the statements in (20) does not need to be there. The possibility that seems to be missing is the last one in (2), that is, (21).

$$(21) \quad \Diamond(\neg Milk \wedge \neg Lactose)$$

Which indicates the situation in which I do not have milk or lactose. The statement that appears to be unnecessary is the last one in (20), that is, (22).

$$(22) \quad (Milk - Lactose) \rightarrow Impossible \langle 1, 0.9 \rangle$$

This is because to derive conclusions within the framework of TMM, as shown in the literature about the latter theory, what is possible is enough. What is impossible is not required. However, to get NAL to make inferences as TMM, (22) is useful and (21) is not necessary. (22) expresses that having milk without lactose is impossible. Therefore, as provided by (21), in a circumstance in which lactose is not had, it is impossible to have milk. We know that we can remove lactose from milk, but NAL only considers the accumulated experience to date (see, e.g., Wang, 2011). If it must think of just (1), the system cannot know that milk without lactose is possible in the real world. In this way, if we assume that (22) is the statement corresponding to (21) in Narsese, we can come to the same conclusions indicated above following TMM. We only need one more assumption: $Possible^E \cap Impossible^E = \emptyset$ (where ' $\cap$ ' and ' $\emptyset$ ' work exactly as in set theory; ' $\cap$ ' is the symbol for intersection, i.e., the symbol referring to the elements that are both in the left set and the right set; ' $\emptyset$ ' expresses empty set), that is, nothing can be both in the extension of *Possible* and in the extension of *Impossible*.

Let us start with the case of Modus Ponendo Ponens. If the first premise is (1), the second premise must be 'I have milk'. Let 'NAL_{TMM}' be the level or module in NAL working as TMM. In NAL_{TMM}, the first premise should be processed as the statements in (20), that is, (22), (23), and (24).

$$(23) \quad (Milk \cup Lactose) \rightarrow Possible \langle 1, 0.9 \rangle$$

$$(24) \quad (Lactose - Milk) \rightarrow Possible \langle 1, 0.9 \rangle$$

The second premise should be translated as (25).

$$(25) \quad Milk \rightarrow Possible \langle 1, 0.9 \rangle$$

From now on, I will ignore the values for f and c . The reason is that in all the statements in Narsese below the values will be the same: $f=1$ and $c=0.9$. I will remove them to simplify.

If (25) is true, NAL_{TMM} must ignore (22), as it can derive (26) from (22).

$$(26) \quad Milk \rightarrow Impossible$$

An objection can be presented here. Contradictions are admitted in NAL. The system can use contradictory statements and come to different conclusions following different paths (e.g., Wang, 2013). Against this, I can say that, while it is correct, we can also implement different levels or modules within NAL that can be at different tiers. Those levels or modules can be mutually independent (e.g., Wang, 2013).

Given that, as I pointed out above (and it can be deduced from Wang, 2013; Theorem 7.4), $(Milk \cup Lactose)^I = Milk^I \cap Lactose^I$ (where ' $\cap$ ' works exactly as in set theory), (25) can be derived from (23), too. But what is relevant is that (27) can be inferred from both (23) and (24).

$$(27) \quad Lactose \rightarrow Possible$$

And this can be understood as an application of Modus Ponendo Ponens.

The following example was that of the derivation of the conclusion in Modus Tollendo Tollens. If (1) is the first premise in an inference of Modus Tollendo Tollens, then the second premise should be 'I do not have lactose', that is, (28).

$$(28) \quad Lactose \rightarrow Impossible$$

In this case, (23) cannot be considered. It allows concluding (27). The same can be said about (24): it also enables to infer (27). Only (22) remains. What (22) allows deriving is (26).

One might think that this does not totally capture the proposal of TMM. According to the latter theory, to apply Modus Tollendo Tollens, it is necessary to make the deliberative effort required to identify (21), and my account seems to ignore that. Nevertheless, this problem is easy to solve. Two different levels or modules could be built, for example, NAL_{TMM1} and NAL_{TMM2} . One of them would work with all the possibilities deliberative effort can note. The other one only would consider the possibilities that can be recovered without effort. In the latter level or module, Modus Tollendo Tollens could not be used. Similar arguments apply to any other example in this paper implying deliberative effort. In fact, this distinction between two levels or modules can help in any circumstance in which effort and lack of effort are related to different expected results within the framework of TMM. In the cases in which the predictions of the theory are different when there is effort and when there is no effort, the building of two diverse levels or modules such as NAL_{TMM1} and NAL_{TMM2} is always a feasible solution.

My next example was that of conditional (3), that is, 'if I go to China, then I go to Beijing', whose possibilities needed to be modulated. Those possibilities could be expressed in Narsese as (29), (30), and (31).

$$\begin{aligned} (29) \quad & (China \cup Beijing) \rightarrow Possible \\ (30) \quad & (China - Beijing) \rightarrow Possible \\ (31) \quad & (Beijing - China) \rightarrow Impossible \end{aligned}$$

In an inference of Modus Tollendo Tollens, the second premise would be 'I do not go to Beijing', which would be in Narsese:

$$(32) \quad Beijing \rightarrow Impossible$$

Given that (29) enables to infer (33),

$$(33) \quad Beijing \rightarrow Possible$$

It must be removed. On the other hand, (31) just allows deducing (32) again. Only (30) can be considered, but it is not useful either. (30) leads to (34).

$$(34) \quad China \rightarrow Possible$$

Which is the opposite of the conclusion in Modus Tollendo Tollens.

Regarding disjunction (5), that is, 'I have milk or I have juice', NAL_{TMM} could attribute statements (35), (36), and (37) to it (if it is understood as inclusive).

- (35) $(Milk \cup Juice) \rightarrow Possible$
- (36) $(Milk - Juice) \rightarrow Possible$
- (37) $(Juice - Milk) \rightarrow Possible$

The second premise in Modus Tollendo Ponens would be ‘I do not have milk’, that is, (38).

- (38) $Milk \rightarrow Impossible$

Statement (38) is incompatible with (35), as the latter enables to come to (25), that is, to $Milk \rightarrow Possible$. The same can be said about (36). (37) can be accepted. It leads to (39).

- (39) $Juice \rightarrow Possible$

That is, to the conclusion in Modus Tollendo Ponens.

In the case of (7), that is, ‘I go to China, or I go to Beijing’, the disjunction requires modulation of its possibilities. In NAL_{TMM} it would be linked to statements (40), (41), and (42).

- (40) $(China \cup Beijing) \rightarrow Possible$
- (41) $(China - Beijing) \rightarrow Possible$
- (42) $(Beijing - China) \rightarrow Impossible$

Statements (40), (41), and (42) are equivalent to, respectively, (29), (30), and (31). If the second premise is now ‘I do not go to China’, it would be expressed as (43).

- (43) $China \rightarrow Impossible$

Thus, (40) should be ignored. It allows coming to (34), that is, to $China \rightarrow Possible$. The level or module could not accept (41) either. NAL_{TMM} can deduce (34) from it, too. Only (42) is coherent with (43). But (42) implies (32), that is, $Beijing \rightarrow Impossible$.

Apparent conjunction (9), that is, ‘you have milk and you feel bad’, can refer to statements (44), (45), and (46).

- (44) $(Milk \cup Bad) \rightarrow Possible$
- (45) $(Bad - Milk) \rightarrow Possible$
- (46) $(Milk - Bad) \rightarrow Impossible$

These statements are related to conjunction (9) because of modulation. As the possibilities of (9) in TMM, statements (44) to (46) correspond to a conditional in natural language (‘if you have milk, then you will feel bad’), not to a conjunction. Therefore, the application of a rule like Modus Ponendo Ponens is viable.

Let us suppose that it is stated that ‘you have milk’, that is, (25), that is, $Milk \rightarrow Possible$. (46) needs to be removed, as it enables to derive (26), that is, $Milk \rightarrow Impossible$. The statements that keep holding are (44) and (45). (47) can be concluded from both.

- (47) $Bad \rightarrow Possible$

The latter statement would be the conclusion in Modus Ponendo Ponens.

Lastly, regarding inference (11), that is, ‘I have milk. Therefore, I have milk or I have juice’, the premise just offers the information in (25), that is, $Milk \rightarrow Possible$. However, the conclusion refers to statements (35) to (37), that is, respectively, $(Milk \cup Juice) \rightarrow Possible$, $(Milk - Juice) \rightarrow Possible$, and $(Juice - Milk) \rightarrow Possible$. From both (35) and (36), NAL_{TMM} can derive that Milk belongs to the extension of

Possible. Nonetheless, an evaluative question applied to the content in (37), that is, *(Juice – Milk) → Possible?*, would give a negative answer and prevent (11) from being accepted.

Conclusions

Hence, it seems that we can build a level or module of NAL such as NAL_{TMM}. That level or module captures the essential inferential processes of TMM by means of a few components of the logic. Beyond the basic inheritance copula, only two of its types of compound terms are necessary: extensional union and extensional difference. The idea of TMM related to the substitution of logical validity for necessity can be considered resorting to evaluative questions. No inference rule of NAL is required.

NAL has much more resources. It can deal with much more details of the conjunctions of possibilities. The literature of TMM cited in the present paper reveals many of those details, which often refer to relations between the clauses. NAL can address them. I have not described above how NAL can do that because the account presented suffices to show the general way NAL can consider the processes proposed by TMM. Maybe this is a point to develop in a future work.

This means that TMM not only can be emulated by means of a non-axiomatic logic such as NAL. It can also be implemented using a computer system such as NARS. This is consistent with the theory. As said, there are already computer programs working as TMM. On the other hand, NAL has the potential to work like, for example, Prolog, a programming language based on classical logic (e.g., Wang, 2013).

If TMM is correct (and, as also indicated, its strong empirical support can lead us to think that), the result of my account can be relevant. If the theory describes the manner our mind acts, a level or module such as NAL_{TMM} can come to the same conclusions as us. I am not claiming that the human mind works following requirements such as those of the inheritance copula, or the concepts of extensional union and extensional difference. What I state is just that, because of NAL_{TMM}, NARS can infer conclusions like those predicted by TMM.

Some challenges need to be addressed, and further research is required. The main point is natural language processing (for general lines to process natural language from NARS, see, e.g., Wang, 2013). We need algorithms to translate expressions in natural language into statements such as those used by NAL_{TMM}. To work in practice as TMM, NAL_{TMM} must be able to store and relate information in a particular way: in the way TMM proposes. Modulation is especially important regarding this. Not all human beings have the same information and knowledge about the same topics. Perhaps, different levels or modules are also necessary to consider the differences in information and knowledge individuals have. This is the path to follow in next studies.

References

- Barres, P. E. & Johnson-Laird, P. N. (2003). On imagining what is true (and what is false). *Thinking and Reasoning*, 9(1), 1-42. <https://doi.org/10.1080/13546780244000097>
- Byrne, R. M. J. & Johnson-Laird, P. N. (2009). “If” and the problems of conditional reasoning. *Trends in Cognitive Science*, 13(7), 282-287. <https://doi.org/10.1016/j.tics.2009.04.003>
- Espino, O. & Byrne, R. M. J. (2013). The compatibility heuristic in non-categorical hypothetical reasoning: Inferences between conditionals and disjunctions. *Cognitive Psychology*, 67(3), 98-129. <https://doi.org/10.1016/j.cogpsych.2013.05.002>
- Espino, O., Byrne, R. M. J., & Johnson-Laird, P. N. (2020). Possibilities and the parallel meanings of factual and counterfactual conditionals. *Memory & Cognition*, 48, 1263-1280. <https://doi.org/10.3758/s13421-020-01040-6>

- Johnson-Laird, P. N. (2010). Against logical form. *Psychologica Belgica*, 50(3/4), 193-221. <http://doi.org/10.5334/pb-50-3-4-193>
- Johnson-Laird, P. N. & Byrne, R. M. J. (2002). Conditionals: A theory of meaning, pragmatics, and inference. *Psychological Review*, 109(4), 646-678. <https://doi.org/10.1037//0033-295X.109.4.646>
- Johnson-Laird, P. N., Byrne, R. M. J., & Khemlani, S. (2023). Human verifications: Computable with truth tables outside logic. *Proceedings of the National Academy of Sciences*, 120(40), e2310488120. <https://doi.org/10.1073/pnas.2310488120>
- Johnson-Laird, P. N., Byrne, R. M. J., & Khemlani, S. (2024). Models of possibilities instead of logic as the basis of human reasoning. *Minds & Machines*, 34(19). <https://doi.org/10.1007/s11023-024-09662-4>
- Johnson-Laird, P. N., Khemlani, S., & Goodwin, G. P. (2015). Logic, probability, and human reasoning. *Trends in Cognitive Sciences*, 19(4), 201-214. <https://doi.org/10.1016/j.tics.2015.02.006>
- Johnson-Laird, P. N. & Ragni, M. (2025). Reasoning about possibilities: Modal logics, possible worlds, and mental models. *Psychonomic Bulletin & Review*, 32(1), 52-79. <https://doi.org/10.3758/s13423-024-02518-z>
- Khemlani, S., Byrne, R. M. J., & Johnson-Laird, P. N. (2018). Facts and possibilities: A model-based theory of sentential reasoning. *Cognitive Science*, 42(6), 1887-1924. <https://doi.org/10.1111/cogs.12634>
- Khemlani, S., Hinterecker, T., & Johnson-Laird, P. N. (2017). The provenance of modal inference. In G. Gunzelmann, A. Howes, T. Tenbrink, & E. J. Davelaar (Eds.), *Computational Foundations of Cognition* (pp. 663-668). Austin, TX: Cognitive Science Society.
- Khemlani, S. & Johnson-Laird, P. N. (2009). Disjunctive illusory inferences and how to eliminate them. *Memory & Cognition*, 37(5), 615-623. <https://doi.org/10.3758/MC.37.5.615>
- Khemlani, S. & Johnson-Laird, P. N. (2022). Reasoning about properties: A computational theory. *Psychological Review*, 129(2), 289-312. <https://doi.org/10.1037/rev0000240>
- Khemlani, S., Orenes, I., & Johnson-Laird, P. N. (2014). The negation of conjunctions, conditionals, and disjunctions. *Acta Psychologica*, 151, 1-7. <https://doi.org/10.1016/j.actpsy.2014.05.004>
- Orenes, I. & Johnson-Laird, P. N. (2012). Logic, models, and paradoxical inferences. *Mind & Language*, 27(4), 357-377. <https://doi.org/10.1111/j.1468-0017.2012.01448.x>
- Quelhas, A. C., Rasga, C., & Johnson-Laird, P. N. (2017). A priori true and false conditionals. *Cognitive Science*, 41(55), 1003-1030. <https://doi.org/10.1111/cogs.12479>
- Quelhas, A. C., Rasga, C., & Johnson-Laird, P. N. (2019). The analytic truth and falsity of disjunctions. *Cognitive Science*, 43(9). <https://doi.org/10.1111/cogs.12739>
- Ragni, M., Kola, I., & Johnson-Laird, P. N. (2018). On selecting evidence to test hypotheses: A theory of selection tasks. *Psychological Bulletin*, 144(8), 779-796. <https://doi.org/10.1037/bul0000146>
- Wang, P. (2006). *Rigid Flexibility: The Logic of Intelligence*. Dordrecht, The Netherlands: Springer. <https://doi.org/10.1007/1-4020-5045-3>
- Wang, P. (2011). The assumptions on knowledge and resources in models of rationality. *International Journal of Machine Consciousness*, 3(1), 193-218. <https://doi.org/10.1142/S1793843011000686>
- Wang, P. (2013). *Non-Axiomatic Logic: A Model of Intelligent Reasoning*. Singapore: World Scientific. <https://doi.org/10.1142/8665>
- Wang, P. (2023). The role of copulas in reasoning. Technical Report #17. Temple AGI Team. Temple University. <https://cis.temple.edu/tagit/publications/TAGIT-TR-17.pdf>