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ALGEBRAIC THINKING IN ELEMENTARY GRADES

POSSIBILITIES WITH BLIND AND NON-BLIND STUDENTS

PENSAMENTO ALGÉBRICO NOS ANOS INICIAIS

Possibilidades com estudantes cegos e videntes

PENSAMIENTO ALGEBRAICO EN PRIMARIA

Posibilidades con estudiantes ciegos y no ciegos

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ABSTRACT

This postdoctoral project aims to investigate aspects of the development of algebraic thinking with blind and non-blind students that can contribute to mathematics teaching practices regarding the inclusion and promotion of learning opportunities in the early years of elementary school. The guiding research question is: "What are the potentialities and challenges of the development of algebraic thinking with blind and non-blind students in the early years of elementary school?". Centered on a qualitative approach, this project will have as a context of data production two classrooms from the early years of elementary school in a public school in the city of Belém, PA, Brazil, in which blind and non-blind students are enrolled. The initial criterion of choosing two classes participating in the research will be to have blind students enrolled in a regular class, as well as the availability of class teachers to discuss the texts and elaborate and develop tasks that involve the development of algebraic thinking. For this purpose, classes will be monitored for three months, considering the school's availability. The data will be produced using the field diary, audio recordings, and video recordings of mathematics classes. During the data analysis, the videos and audios will be revisited several times, seeking to identify elements that help us respond to the research objective, and these will be transcribed and analyzed. We hope that the results of this study will contribute to the construction of inclusive educational practices in the early years of elementary school that enable blind and non-blind students to understand the fundamental concepts involved in the development of algebraic thinking.

Keywords: early algebra. elementary school. inclusion. blind students. mathematics education.

RESUMO

Este projeto de pós-doutorado tem como objetivo investigar aspectos do desenvolvimento do pensamento algébrico com estudantes cegos e videntes que possam contribuir para as práticas de ensino de matemática no que diz respeito à inclusão e promoção de oportunidades de aprendizagem nos anos iniciais do Ensino Fundamental. A questão norteadora da pesquisa traduz-se por: "Quais as potencialidades e os desafios do desenvolvimento do pensamento algébrico com estudantes cegos e videntes nos anos iniciais do Ensino Fundamental?". Centrado em uma abordagem qualitativa, este projeto terá como contexto de produção dos dados duas salas de aula dos anos iniciais do Ensino Fundamental de uma escola pública de Belém, PA, Brasil, em que estejam matriculados estudantes cegos e videntes. O critério inicial de escolha duas turmas participantes da pesquisa será possuir estudantes cegos matriculados em uma turma regular, bem como, a disponibilidade das professoras das turmas em discutir os textos e elaborar e desenvolver tarefas que envolvam o desenvolvimento do pensamento algébrico. Com este propósito, suas aulas serão acompanhadas durante três meses, considerando-se também a disponibilidade da escola. Os dados serão produzidos com a utilização do diário de campo, de audiogravações e de videogravações das aulas de matemática. Durante a análise de dados os vídeos e audios serão revisitados diversas vezes buscando identificar elementos que nos ajudem a responder o objetivo da pesquisa, estes serão transcritos e analisados. Esperamos que os resultados deste estudo contribuam para a construção de práticas educativas inclusivas nos anos iniciais do Ensino Fundamental que possibilitem a estudantes cegos e videntes a compreenderem os conceitos fundamentais envolvidos no desenvolvimento do pensamento algébrico.

Palavras-chave: pensamento algébrico. anos iniciais. inclusão. estudantes com cegueira. educação matemática.

RESUMEN

Este proyecto postdoctoral tiene como objetivo investigar aspectos del desarrollo del pensamiento algebraico en estudiantes ciegos y no ciegos que puedan contribuir a las prácticas de enseñanza de las matemáticas en cuanto a la inclusión y promoción de oportunidades de aprendizaje en los primeros años de la escuela primaria. La pregunta rectora de la investigación es: ¿Cuáles son las potencialidades y desafíos del desarrollo del pensamiento algebraico en estudiantes ciegos y no ciegos en los primeros años de la escuela primaria? Centrado en un enfoque cualitativo, este proyecto tendrá como contexto de producción de datos dos aulas de los primeros años de la escuela primaria de una escuela pública de la ciudad de Belém, PA, Brasil, en las que están matriculados estudiantes ciegos y no ciegos. El criterio inicial para elegir dos clases participantes en la investigación será tener estudiantes ciegos matriculados en una clase regular, así como la disponibilidad de profesores de clase para discutir los textos y elaborar y desarrollar tareas que involucren el desarrollo del pensamiento algebraico. Para ello, se realizará un seguimiento de las clases durante tres meses, considerando la disponibilidad del colegio. Los datos se producirán utilizando el diario de campo, grabaciones de audio y grabaciones de video de las clases de matemáticas. Durante el análisis de los datos, los videos y audios serán revisados varias veces, buscando identificar elementos que nos ayuden a responder al objetivo de la investigación, y estos serán transcritos y analizados. Esperamos que los resultados de este estudio contribuyan a la construcción de prácticas educativas inclusivas en los primeros años de la escuela primaria que permitan a estudiantes ciegos y no ciegos comprender los conceptos fundamentales involucrados en el desarrollo del pensamiento algebraico.

Palabras clave: pensamiento algebraico. escuela primaria. inclusión. estudiantes ciegos. educación matemática.

Why critical mathematics education, blind students, and algebraic thinking?

> It was dusk, we were walking along Senador Lemos Avenue, towards D. Pedro I Square, in Belém, Brazil. We were about to cross a corner, Sônia was guiding me; suddenly, I realized she was about to cross when a car approached us. I hurriedly said: "Can't you see the car coming? If you want to die, go alone. I still have a lot of life to live." Surprised, she asked: "Wow, how did you know a car was coming?" "Well, I orient myself by parallel traffic." "How does that work?" "I identify the flow of cars based on the intensity of the sound they make, so I can know their direction and how close they are, which gives me more security [...] I can no longer consider vision as my main sense because, as a moving body, I move through the streets, I think, live and dream, I use the perceptive sensitivity heightened by everything that happens in the world. In this way, I feel this world through the sounds my ears can capture, the smells things emanate, and the nuances of the air when they touch my skin. In addition to perception, I also count on Scarlett, my cane, which integrates my motor system." Sônia was delighted with my explanation. She, who had never stopped to think about this different way of being and existing in the world, suddenly found herself guided by her blind friend (Carvalho, 2021, p.18-19, our translation).

People perceive themselves, others, and the world in different ways. In this excerpt from the chronicle "Conversas de Rua" (Street Talks), Mônica de Nazaré Carvalho uses, among other things, her auditory perception to move around safely. The sound she captures is essential in her decisions to stop or move. This narrative teaches us the importance of the different senses in understanding the world. It inspires us to reflect on the importance of blind children being able to encounter mathematical knowledge inside and outside the classroom using their different senses. Thus, this project is based on the following guiding question: What are the potentialities and challenges of developing algebraic thinking with blind and non-blind students in the early years of elementary school?

When we start a project, some questions arise: What is the appropriate theoretical framework for this research? What lenses should be used to look at the object of study? When we asked ourselves these questions during the elaboration of this project, we opted for the scientific rigor of the epistemological perspective, by the methodological possibilities of envisioning what is not but could be, but mainly and above all, by the feeling that remains with us after reading a text by Skovsmose (2016) and collaborators: the feeling of the possibility of joining a collective effort for the construction of a fairer and inclusive society. In this sense, this project will use critical mathematics education as a theoretical reference.

Critical mathematics education is concerned with inclusion education, as everyone must have access to knowledge, specifically mathematical knowledge. Such knowledge is vital for students from vulnerable groups to understand the world and have the possibility to progress academically. In addition to this epistemological aspect, the encounter between different students in a dialogic and cooperative learning environment helps all students learn to respect different perspectives, understand the other as a source of knowledge, and learn to work in groups, which are essential aspects for life in a democratic society (Skovsmose, 2016).

Skovsmose (2016) highlights the importance of research in mathematics education, curriculum, and pedagogical practices in the classroom, at all levels of teaching, to contemplate different groups of students, including students with disabilities. From this perspective of the importance of the democratization of mathematical knowledge for different groups of students, this project is dedicated to blind students in an inclusive environment where blind and non-blind students meet and learn together.

Marcone (2015) accentuates that blind students are generally not associated with learning the different mathematical contents of the curriculum but only with a part of them. Thus, it becomes essential that research that embraces mathematics education and society challenges such perspectives (Skovsmose, 2016) and addresses teaching and learning for blind students, focusing on the different contents and ways of thinking mathematically, among them the algebraic thinking.

Moses (2001) draws attention to the fact that some mathematical knowledge is decisive in the academic progress of underrepresented groups of students, and among them, the author emphasizes the decisive role of algebra. The international (NCTM, 2000) and national (Brazil, 2018) curriculum guidelines also highlight the importance of working on algebraic thinking from the beginning of schooling.

Lins and Kaput (2004, p. 48) define algebraic thinking in the context of early algebra as:

> Acts of deliberate generalization and expression of generality... [and] reasoning based on the forms of syntactically-structured generalisations, including syntactically and semantically guided actions. This characterisation of the broad kinds of algebraic reasoning helps us then discuss forms of algebraic thinking appropriate for young children and the conditions that may promote them.

This definition of algebraic thinking meets this research by seeking to enable the development of algebraic thinking in the context of childhood. According to Nacarato and Custodio (2018, p. 27, our translation):

> While algebra can be thought of as a language, as a particular type of mathematical activity, algebraic thinking is a set of intellectual skills necessary for algebra (thinking analytically, generalizing, abstracting, etc.).

According to the authors, the experiences with algebraic thinking at the beginning of schooling should be based on multiple languages, problematization and questioning, lucidity, imagination, the use of the body as a whole, and the use of manipulated materials.

Fernandes and Healy (2013) contribute to understanding the development of algebraic thinking in students with disabilities. The authors give visibility to forms of generalizations expressed by deaf and hearing students in the final years of elementary school. The study results bring evidence of generalizations expressed mainly by sign language, as well as a tendency of students to look for ways to relate two variables.

Uliana (2013) developed a pedagogical kit that favors tactile exploration in teaching and learning concepts related to plane geometric figures and polynomial function graphs. The author highlights the importance of tactile exploration in teaching and learning the mentioned concepts.

Marcelly (2015) highlights the importance of three essential aspects for teaching and learning mathematics in classrooms where students with and without disabilities are present. The first consists of constructing manipulative materials for mathematics teaching from the perspective of the universal design for learning (UDL) teaching approach. The second highlights the importance of working conditions in schools, and the third is teacher education. The author emphasizes that teaching mathematics from an inclusive perspective is intentional planning and involves constructing manipulative materials accessible to everyone present in the classroom: blind and non-blind students. During the research, materials that favored tactile exploration were created to work on different mathematical contents of different themes, including trigonometry, spatial geometry, plane geometry, and arithmetic operations in middle school.

The research developed by Uliana (2013) and Marcelly (2015) has in common the essential appreciation of the specificities of blind students and students with visual impairments and the proposition of materials and pedagogical practices that go beyond the visual exploration of mathematical concepts.

Filha, Ribeiro, and Santos (2022) investigated the development of algebraic thinking in the early years of elementary school with visually impaired students. The research participants attended regular school in one part of the day, and in the after-school period, they attended a rehabilitation and support center for students with visual impairment. The authors' interventions were developed in the second space and consisted of developing a didactic sequence. The results prove that the students acquired the concepts referring to equality and inequality. The authors also highlight the importance of children's literature, playfulness, manipulative materials, and sensory books for the development of algebraic thinking in elementary school.

Although there is an expressive number of investigations related to inclusion in the field of mathematics education (Barros, 2021; Kollosche *et al.*, 2019; Marcelly, 2015; Marcone, 2015; Filha, Ribeiro, & Santos, 2022; Fernandes, & Healy, 2010; 2013; Sales, 2015; Sales, Penteado, & Moura, 2020; Uliana,

2013), none of them present an explicit connection with the development of algebraic thinking for blind and non-blind students in the early years of regular elementary school.

In the next section, we discuss the development of algebraic thinking in the early years of elementary school.

Algebraic thinking in the early years of elementary school

The development of algebraic thinking in the early years of elementary school involves work with generalized arithmetic, functional thinking, and modeling and is centered on generalization and the expression of generalization through different languages, such as natural language and pictorial representation (Kaput, Carraher, & Blanton, 2008). In this sense, the focus is not on the content of algebra, anticipating it in the early years in the same way as it was taught in the final years of elementary school, but rather on a way of thinking algebraically based on three principles: "It builds heavily on background contexts of problems. It only gradually introduces formal notation. And, it is tightly interwoven with the following topics from early mathematics curriculum" (Carraher, Schliemann, & Schwartz, 2008, p. 236). Luna, Souza, and Souza (2015), Nacarato and Custódio (2018), Vergel (2021), Nacarato and Custódio (2018), and Radford (2011, 2013, 2022a) work with the development of algebraic thinking in the early years of elementary school.

Nacarato and Custódio (2018) emphasize that when exploring patterns with children, we must address vocabulary related to algebraic thinking, such as sequence and regularity. They emphasize the possibility of using terms such as pattern, motif, or secret to refer to elements of a sequence that repeat themselves. When working with pre-established repetitive sequences, the authors emphasize the need to present the repeated pattern at least two and a half times to allow children to identify it. Playfulness, the use of the body and movement, musical resources, manipulative materials, recording (oral, written, pictorial, photographic, and videographic), questioning, and problematization are fundamental aspects in the development of algebraic thinking at this level of education (Nacarato, & Custódio, 2018).

Luna, Souza, and Souza (2015) indicate that children in the early years can produce algebraic discourses, which favors the involvement of these students with Algebra at other levels of education. One aspect that favors children's production of multimodal algebraic discourses is the teacher's openness, which creates rich opportunities for children to express themselves using their bodies, natural language, written records, and pictorial records. During the tasks described by the authors, the children made movements that expressed the pattern of a repetitive sequence, which the authors called embodied algebraic discourses. According to Luna, Souza, and Souza (2015), producing embodied algebraic discourses creates favorable conditions for children to create other types of discourse, such as written algebraic discourse. The authors also highlight the importance of interaction between children and between the teacher and children for the production of algebraic discourses in the early years.

According to Radford (2013), the generalization of patterns involves identifying a common characteristic in some elements of the sequence, which is called commonality, generalizing this commonality to all elements of the sequence, and using this commonality to determine any term of the sequence.

Radford (2010, 2011, 2022b), Moretti and Radford (2021), and Vergel (2021) contribute to our reflection on the importance of corporeality in the teaching and learning processes that involve the development of algebraic thinking in the early years of Elementary School. According to Vergel (2021), students' cognitive processes are constituted by tactile, perceptive, and synesthetic sensory modalities, among others. During tasks focused on the development of algebraic thinking through the exploration of patterns, students in the early years mobilize semiotic resources such as written and spoken language, gestures, and actions (Vergel, 2021).

Based on the recognition of the development of algebraic thinking in the initial years of elementary school, in the next section, we present the objectives of the project.

Objectives of the study

In order to answer the research question, this project aims to investigate aspects of the development of algebraic thinking of blind and non-blind students that can contribute to mathematics teaching practices regarding the inclusion and promotion of learning opportunities in the early years of elementary school. From this general objective, the following specific objectives are outlined:

1. Identify and describe pedagogical practices that incorporate the development of algebraic thinking in the early years;

2. Identify and describe the potentialities and challenges of the development of algebraic thinking with blind and non-blind students during the mathematics classes of the teachers participating in the research;

3. Describe and analyze aspects of the pedagogical practices that enhance the development of algebraic thinking in an inclusive learning environment.

In the next section, we will address the methodological approaches of the project.

Methodology

Focused on a qualitative approach (Bogdan; Biklen, 1994), this project will have as a context of data production two classrooms from the early years of elementary school of a public school in the city of Belém, PA, Brazil, in which blind students are enrolled in a regular class. This aspect becomes essential since we seek to develop research in a regular environment intended not only for blind students but in an environment that is a meeting space between blind and non-blind students. This research does not aim to compare the performances of blind and non-blind students. Instead, we seek to understand, for instance, teaching and learning strategies that consider the specificities of blind students and contribute to the development of algebraic thinking of all students in the classroom.

At the time of this publication, the two classes have already been identified: the fourth and fifth grades of elementary school in two public schools in Belém. The fourth-grade class is part of a state school and comprises 29 students aged between 8 and 10. One of the students is a girl with congenital blindness who is literate and learning Braille. Congenital blindness is a condition acquired from birth. “Most children with visual impairment are born with this condition as a result of congenital or hereditary diseases” (Bruno, 2006, p. 13, our translation).

The fifth-grade elementary school class is part of a municipal school and has 26 students aged between 9 and 12. One of them is a boy with adventitious blindness acquired throughout life (Bruno, 2016; Nascimento; Nascimento, 2020). According to Bruno (2006, p. 13, our translation), “If a child becomes visually impaired after the age of five, they will have already developed practically all their visual potential and will be able to preserve images and visual memory.”

The initial criteria of choice of the two classes participating in the research will be to have blind students in a regular class, as well as the availability of class teachers to discuss the texts and elaborate and develop tasks that involve the development of algebraic thinking. For this purpose, the classes will be monitored for three months, and the availability of the school will be considered. The data will be produced using the mathematics classes’ field diary, audio and video recordings, tasks done in class, and all manipulatives used by students. During the data analysis, the videos and the audios will be revisited several times to identify critical events (Powell, 2015; Powell; Francisco; Maher, 2004). Such events will be transcribed and analyzed.

In the next section, we address possible contributions from the research being developed for this project.

Expected outcomes or contributions

The initial phase of this two-year postdoctoral project aims to investigate aspects of the development of algebraic thinking in blind and non-blind students that can contribute to mathematics teaching practices regarding the inclusion and promotion of learning opportunities in the early years of elementary school.

Up until the publication of this paper, researchers had prepared tasks that sought to explore patterns through touch and hearing. They contacted the schools where data production would be carried out and established a partnership with the two teachers of the classes that would participate in the research.

Mathematics teaching and learning has been predominantly based on visual perception (Nascimento; Nascimento, 2020). When investigating the representation of blind young people in school inclusion, Nascimento and Nascimento (2020) highlight that they presented ambiguous representations regarding inclusion. Some young people reported situations related to inclusion, but some related to exclusion based on their school experiences. For example, the blind young people highlighted that the teacher plays an essential role in the inclusion process in mediation and that many of the teachers they worked with learned Braille, playing a decisive role in the school inclusion of these students. At the same time, they emphasize that they often could not learn mathematical content due to the lack of accessibility in the classes. A young blind student narrates her experiences in mathematics classes:

> [...] They didn't know, especially those in Exact Sciences, they didn't know how to behave in a situation of how to teach a blind student, so what happened?... The teacher was giving a class, and let's say, on a function of the first degree, instead of saying something like: there's an x, there's a y, let's move an x here, no, he said something like, let's move this guy here, this other guy there, he didn't describe what was on the board. I had a big problem... teacher, tell me what's written, I don't know what... he didn't do it, you know? So, I'm not saying that I needed a supplementary class, but I needed sensitivity on his part to understand the situation, right? (Nascimento & Nascimento, 2020, p.171-172).

The student's report provides evidence that the communication between the teacher and the students in mathematics class is often based on images. In this case, the interpretation of the image without audio description does not consider the specificities of the blind student. Nascimento and Nascimento (2020) also highlight the importance of using audio description as methodological resources for explanations that are based on images; access to mathematics subject content; the availability of Braille or digital text on the notebook are essential structural aspects in the school inclusion of blind students.

Critical mathematical education (Skovsmose, 2000, 2001, 2007, 2014, 2023) has examined the classroom and how mathematical content has been taught historically. By bringing concepts such as the ideology of certainty and the paradigm of exercise, it points out criticisms of pedagogical practices in mathematics classes and invites the community of educators and mathematics researchers to invest in new paths.

Critical mathematical education expresses itself through an invitation to new narratives in the mathematics classroom. Thus, inspired by this perspective, we consider that there is a space for narratives and pedagogical practices that do not hierarchize sensory perceptions in the process of teaching and learning mathematics or that do not limit the definition of student (un)success concerning the possibility of seeing in the mathematics classroom.

The tasks were developed following the following aspects. Tasks should be:

> (i) Accessible to all students: This aspect is based on providing a learning environment in mathematics classes that is accessible to all students, in which all students, regardless of their differences, come together in the classroom to learn together (Penteado & Skovsmose, 2022;

Skovsmose, 2016). This aspect helps students in the early years of elementary school recognize that there are different ways of learning mathematics and that they are all important and valuable. A learning environment with these characteristics takes into account aspects of diversity and allows students to recognize the ways that help them think mathematically and to respect and be interested in the ways used by other people. Thus, the tasks designed seek to value the different possibilities of learning mathematics and allow blind and non-blind students to perform the same task.

(ii) Allow exploration of patterns through touch and hearing: The auditory and tactile sensory modalities are central elements for the identification of patterns; therefore, materials that contribute to such exploration, such as manipulable materials (sticks) and objects that produce sound (whistles and xylophones), were selected.

(iii) Isolate the attribute used to identify the pattern in the manipulative material or the sound resource. For example, when creating an activity with pattern identification based on the attribute of the shape of the objects, this attribute should be isolated so as not to compete with other attributes such as color. Let us imagine that in a specific activity, blind and non-blind students have a sequence of cubes and pyramids. In this way, the main attribute for identifying the pattern would be the shape of the objects. Both blind and sighted students could perform the activity. However, if the cubes were green and the pyramids were yellow, thus having two different attributes for forming the pattern, the activity would not be focused only on the shape since non-blind students could identify the pattern not by the shapes but by the colors green and yellow. In this case, it is important that it is isolated when trying to work on the attribute of shape as a defining element of a sequence pattern. This aspect will help all students direct their attention to the shape attribute.

(v) Promote dialogue and cooperation: Tasks should be designed and organized in such a way as to enable them to be developed in pairs or groups, favoring the process of dialogue among students. Favoring cooperative and dialogical work among students becomes essential in an inclusive environment in the early years of elementary school and finds justification in the context of teaching and learning mathematics, as well as in the social context for learning to live with others (Alrø & Skovsmose, 2004; Faustino & Skovsmose, 2020; Faustino, Moura, & Milani, 2024; Freire, 2005, 2014; Skovsmose, 2023, 2024). Dialogue is an essential aspect of a learning environment in the early years of elementary school (Faustino, 2018). It contributes to the negotiation of mathematical meanings (Mengali, Passos, Nacarato, 2017) and becomes essential for the process of generalization, with natural language being a recognized way to express generalizations (Luna, Souza, Souza, 2015; Nacarato, Custódio, 2018; Moretti, Radford, 2021). In the social context, dialogue becomes essential for students to learn to live with and respect others, to actively listen and learn from differences.

(vi) Guarantee students' right to mathematics learning: The tasks were designed to address the thematic unit of algebra present in the Base Nacional Comum Curricular (BNCC) (Brasil, 2018) and to provide opportunities for the development of algebraic thinking. To this end, we seek to provide accessible tasks in which students, regardless of their differences, can: 1) Identify patterns in a pre-established sequence; 2) Describe patterns; 3) Continue the sequence; 4) Create sequences with a pattern; 5) Create ways to determine any element of the sequence.

After being elaborated by the researchers, the tasks were discussed with the community of blind researchers and teachers, seeking to ensure their accessibility. In the data production phase starting in the second half of 2023, these were discussed with the class teachers, and new tasks were developed with them.

We hope this project contributes to the identification and construction of inclusive pedagogical practices (for instance, tasks, school environment organization, and manipulative materials) that consider the

specificities of blind students and contribute to the development of algebraic thinking with blind and non-blind students in the early years of elementary school.

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