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# LA VERDAD Y LO BELLO

## EN TEORÍAS AXIOMÁTICAS SIGNIFICATIVAS EN EL CONTEXTO DE CRITERIOS OBJETIVOS DE BELLEZA CIENTÍFICA

*TRUTH AND THE BEAUTIFUL*

*In meaningful axiomatic theories in the context of objective criteria of scientific beauty*

*A VERDADE E O BONITO*

*Sobre teorias axiomáticas significativas no contexto de critérios objetivos de beleza científica*

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## ABSTRACT

The article examines the problem of identifying truth and beauty in meaningful axiomatic theories, and the formation of these values in the process of teaching certain sections of a mathematics course of a general education school with the characteristic features of meaningful axiomatic theories. Certain groups of criteria of scientific or mathematical beauty are considered as a factor in the formation and identification of truth and beauty. Such criteria were introduced by the Scottish philosopher Hutcheson in the 18th century, and his many followers in subsequent centuries. This work examines the formative, unifying and logical groups of objective criteria of scientific beauty. It shows that: a) formative criteria: symmetry, comparison, harmony, rhythm, applicability are more prone to the formation and identification of the beauty of objects of a meaningful axiomatic theory than the truth in it; b) unifying criteria - unity of diversity, generality, mathematical recording of scientific laws are aimed at revealing the truth in a meaningful axiomatic theory. And the beauty of objects here is determined by the connection of criteria with truth, c) logical criteria - logical rigor, clarity, simplicity, reduction of the complex to the simple are aimed at the formation and identification in a meaningful axiomatic theory of both truth and the formation and identification of the beauty of objects.

Keywords: process of teaching mathematics. truth. beauty. scientific beauty. axiomatic theory.

## RESUMEN

El artículo examina el problema de identificar la verdad y la belleza en teorías axiomáticas significativas, y la formación de estos valores en el proceso de enseñanza de ciertas secciones de un curso de matemáticas de una escuela de educación general con los rasgos característicos de las teorías axiomáticas significativas. Ciertos grupos de criterios de belleza científica o matemática se consideran un factor en la formación e identificación de la verdad y la belleza. Dichos criterios fueron introducidos por el filósofo escocés Hutcheson en el siglo XVIII y sus numerosos seguidores en los siglos posteriores. Este trabajo examina los grupos formativos, unificadores y lógicos de criterios objetivos de belleza científica. Muestra que: a) los criterios formativos: simetría, comparación, armonía, ritmo, aplicabilidad son más propensos a la formación e identificación de la belleza de los objetos de una teoría axiomática significativa que la verdad en ella; b) criterios unificadores: unidad de diversidad, generalidad, registro matemático de leyes científicas tienen como objetivo revelar la verdad en una teoría axiomática significativa. Y la belleza de los objetos aquí está determinada por la conexión de los criterios con la verdad, c) los criterios lógicos: el rigor lógico, la claridad, la simplicidad, la reducción de lo complejo a lo simple tienen como objetivo la formación e identificación en una teoría axiomática significativa de ambas verdades. y la formación e identificación de la belleza de los objetos.

Palabras clave: proceso de enseñanza de las matemáticas. verdad. belleza. belleza científica. teoría axiomática.

RESUMO

O artigo examina o problema de identificar a verdade e a beleza em teorias axiomáticas significativas e a formação desses valores no processo de ensino de certas seções de um curso de matemática de uma escola de educação geral com os traços característicos de teorias axiomáticas significativas. critérios de beleza científica ou matemática são considerados um fator na formação e identificação da verdade e da beleza. Tais critérios foram introduzidos pelo filósofo escocês Hutcheson no século XVIII e por seus muitos seguidores nos séculos subsequentes. Este trabalho examina os grupos formativos, unificadores e lógicos de critérios objetivos de beleza científica. Mostra que: a) critérios formativos: simetria, comparação, harmonia, ritmo, aplicabilidade são mais propensos à formação e identificação da beleza dos objetos de uma teoria axiomática significativa do que a verdade nela contida; b) critérios unificadores - unidade de diversidade, generalidade, registro matemático de leis científicas visa revelar a verdade em uma teoria axiomática significativa. E a beleza dos objetos aqui é determinada pela conexão dos critérios com a verdade, c) critérios lógicos - rigor lógico, clareza, simplicidade, redução do complexo ao simples visam a formação e identificação em uma teoria axiomática significativa de ambas as verdades e a formação e identificação da beleza dos objetos.

Palavras-chave: processo de ensino da matemática. verdade. beleza. beleza científica. teoria axiomática.

## Introduction

It is difficult to overestimate the role of general education in the formation and development of two very important human values: truth and beauty. Each school subject in its own way participates in the formation of these key components of the student's value system. In general, truth and beauty are clearly manifested and are closely related to each other, especially in mathematics. The role of mathematics in the formation, discovery and confirmation of truth is well known. At the same time, as von Neumann, one of the fathers of cybernetics, noted, aesthetic motives played an important role in the development of mathematics. Truth and beauty are especially closely interrelated and are especially noticeable in axiomatic theories or in those branches of mathematics where the axiomatic method is used.

As N. Bourbaki notes: “It was this “deductive” mathematics that formed the basis for the development of philosophical and mathematical thought in subsequent centuries” (Bourbaki N., 1963, p. 10). And the subsequent development of mathematical thought is aimed at identifying and confirming the truth, and one of its main tools is the axiomatic method. Beauty in science or scientific beauty was introduced by the 18th century Scottish philosopher F. Hutcheson, who presented standards of beauty as criteria: a mathematical object is beautiful if it satisfies any of the criterion he established. Hutcheson proposed three such criteria: unity of diversity, community and knowledge of non-obvious truths (F. Hutcheson, 2004). Later, mathematicians, philosophers, physicists, psychologists and methodologists used many other similar criteria to determine the aesthetics of both mathematics and the process of teaching it (see G.I. Sarantsev 2000, M.A. Rodionov, E.V. Liksina 2003). In the works of H.S. Mikaelian 2015, H.S. Mikaelian 2019) these criteria are classified according to objective and subjective characteristics, which in turn are divided into groups. When dividing objective criteria, groups of formative, unifying and logical criteria are obtained.

The axiomatic method and the creation of corresponding theories went through three periods of development. In connection with the possibilities and practice of application in general education, in this work we consider only meaningful axiomatic theories. The role of each of the formative, unifying and logical groups of criteria of scientific beauty in the formation of truth and beauty in meaningful axiomatic theories is discussed. In addition, Euclid's Elements (Euclidean Elements, 1948), a high school geometry course, a linearly ordered field of real numbers and a partially ordered field of rational fractions (Laszlo Fuchs, 2011) are considered as examples of meaningful axiomatic theory.

In all these theories, the formative, unifying and logical groups of objective criteria of scientific beauty play different roles in the process of identifying and forming the true and beautiful. For formative groups of criteria, the beautiful is primary, for unifying groups of criteria - the true, the logical groups of criteria - both the true and the beautiful.

## 1. What is axiomatic theory?

An axiomatic theory is a scientific theory based on: a) some initial concepts without definition, through which the remaining concepts of t theory are defined, b) some initial provisions or true propositions, called axioms, and the remaining provisions or true propositions of the theory, called theorems, are derived by simple logical techniques from these axioms (Mendelssohn Elliot, 2015).

The path of discovering truth through the construction of an axiomatic theory is called the axiomatic method. The axiomatic method is considered the most important way to discover scientific truth. For this reason, various scientists have tried to apply it in research in the fields of philosophy, economics, biology, sociology. However, the method found full application primarily in mathematics, where it was and is considered the main way to discover the truth.

The axiomatic method in mathematics had three periods of development. The first or initial period of development of the method is associated with the names of the founders of geometry, Thales and Pythagoras, who laid the foundations of the axiomatic approach, although the method was fully manifested only in Euclid’s Elements (Elements of Euclid, 1948). However, "Elements" had a number of shortcomings. They were especially noticeable when it came to such basic ideas as equality, continuity, clarity of definition of concepts, etc. Despite the shortcomings, the Euclidean understanding of the axiomatic method in mathematics survived until the beginning of the 19th century. Here the axiomatic theory appeared in a meaningful manifestation, that is, its provisions were associated with a certain group of objects, and the axioms were considered indisputable judgments or truths about the objects of this group.

The second period of development of the axiomatic method is associated with the discovery of non-Euclidean geometry. At the beginning of the 19th century, Lobachevsky and Boya showed that, along with the axiom of parallelis, Euclid’s axiomatic theory has the right to exist and a position that denies it.

At the same time, along with non-Euclidean geometry, axiomatic theory of natural numbers /Peano/, axiomatic set theory /Zermelo/, axiomatic geometry /Hilbert/ and other axiomatic theories arose (A.A. Frenkel, I. Bar-Hillel, 1973) . However, these constructed axiomatic theories differed from the theories constructed during the first or meaningful axiomatic theories. In them, axioms were considered only as preliminary provisions of the theory, and the question of their truth was related to the model or interpretation. Moreover, if the model of a meaningful axiomatic theory was the only one, then here the same theory could have different models. And faith in the truth obtained by an axiomatic theory constructed in this way was conditional in nature and also depended on the nature of the theory model. It is worth noting that, for example, faith in non-Euclidean geometry began to form after the appearance of the model proposed by Felix Klein for this theory.

The third period of development of the axiomatic method is associated with the problem of clarifying the logical concept of inference (Elliot Mendelson, 2015). On the way to realizing this problem, a formal axiomatic theory or mathematical logic was created. In a formal axiomatic theory, the alphabet of this theory is first given, consisting of certain sets or groups of subject symbols, symbols of operations and relations. The rules with the help of which expressions or terms of the theory are obtained using symbols of operations are noted. From terms, through symbols of relations, formulas of the theory are obtained, some of which are called axioms. Rules for the derivation of the theory are also given, allowing one to obtain or derive other formulas from some formulas. Under these conditions, the conclusion of a theory is a finite sequence of formulas if each of its terms is either an axiom or is derived from its previous formulas. The last derivation formula is called a theorem. Thus, both axioms and theorems of formal axiomatic theory are finite sequences of symbols, devoid of any meaning, and acquiring meaning only on the model. Here questions arise about the isomorphism of different models of the same formal axiomatic theory, the truth of images of theorems in models and other similar questions, the solutions of which represent the best achievements of mathematical logic (Elliot Mendelson, 2015).

In a school mathematics course, only meaningful axiomatic theories or their individual parts are considered. If a geometry course in different countries (including Armenia) is taught as a separate academic subject, then it, in fact, represents various modifications of the meaningful axiomatic theory of Euclidean geometry.

In the case of algebra the situation is more complicated. Here, there are significant deviations in the choice of content for a general education course, even among textbooks written according to the same educational standard of the same country. In terms of including meaningful axiomatic theory, the high school algebra standard of the Republic of Armenia, for example, makes it possible to present topics on real numbers, rational fractions and inequalities, which are key to the entire course, from the perspective of meaningful axiomatic theory. In essence, here we have meaningful axiomatic theories of a linearly ordered field of real numbers and a partially ordered field of rational fractions or models of axiomatic theories of linearly and partially ordered fields (L. Fuchs, 2011). The corresponding presentation is implemented in textbooks (G. S. Mikaelyan, 2006, 2007, 2008). But without a system of axioms, introducing real numbers only through infinite decimal fractions, especially performing arithmetic operations with them, which is carried out in the Russian textbook (S.N. Nikolsky and others, 2014), is an extremely difficult process, probably insurmountable for a high school student, moreover, deprived of the necessary motivational basis.

## 2. Objective criteria of mathematical beauty

As noted in the introduction Iin the works H. S. Mikaelian, 2015, 2019, the criteria of scientific beauty are divided by objective and subjective nature. Here we will consider only objective criteria - criteria that are inherently properties of objects in different fields of science. These are symmetry, harmony, logical rigor, etc. Symmetry, for example, is a property of various objects in mathematics, physics, chemistry and other natural sciences, logical rigor is a property of scientific thought, etc.

In the same works H. S. Mikaelian, 2015, 2019, the objective characteristics of scientific beauty are divided into three groups. The first group includes the constituent elements of nature: symmetry, comparison, harmony, rhythm, applicability. These are the so-called formative signs of scientific beauty. The second group of objective criteria includes the unity of diversity, generality, mathematical notation of scientific laws, etc. Such criteria conceptually unite scientific objects. Therefore, this group is called a group of unifying characteristics. The third group of objective criteria relates to scientific language. This is logical rigor, clarity, simplicity, reduction of the complex to the simple. They are called the logical signs of scientific beauty.

In the works H.S. Mikaelian, 2015, 2019 the role of formative, unifying and logical criteria in identifying and forming beauty in the process of teaching mathematics was studied in detail.

## 3. Manifestations of he formative criteria of scientific beauty in meaningful axiomatic theories

How do these groups of criteria manifest themselves in axiomatic theories? The emergence of both truth and beauty in axiomatic theories is of a hereditary nature. First of all, they are determined by the truth and beauty of the system of axioms and rules of inference. In a meaningful axiomatic theory, the problem is also related to the nature of the objects studied in this theory. Let us consider the manifestations of formative criteria in meaningful axiomatic theories.

In meaningful axiomatic theories we are talking about a group of specific objects, and the forming criteria are of an objective nature. And it is natural that symmetry, comparison, harmony and rhythm can be more vivid and expressive in these objects than in the axioms related to them. For this reason, formative criteria tend to express more the beauty of the axioms of a meaningful axiomatic theory than the truth. As for theorems, they are the most vulnerable elements of a meaningful axiomatic theory, since the element of proof here is incomplete. In addition, the system of axioms expressing the essence of observed objects remains largely incomplete. As a result, the problem of the truth of the theorem again suffers, and the question of beauty is reduced to checking the heredity of the forming properties, which is basically preserved.

As an example of a meaningful axiomatic theory, consider the geometry of Euclid according to the “Elements” (Euclid, 1948). In the Elements, Euclid considers nine axioms [7]. They all express truths. But they are too few to express the true state of affairs in Euclidean geometry. For this reason, there is not very much faith in the truths or proofs expressed in the Elements. However, this system of axioms very well satisfies the formative signs of scientific beauty, as shown in the following table.

| N | axiom | aesthetic criteria |
|---|---|---|
| 1 | equal to the same are equal to each other | harmony |
| 2 | if equals are added to equals, then equals are obtained | harmony<br>symmetry<br>comparison |
| 3 | if equals are subtracted from equals, then the remainders will be equal | harmony<br>symmetry<br>comparison |
| 4 | If equals are added to unequals, then unequals are obtained. | harmony<br>comparison |
| 5 | doubles of the same thing are equal to each other | comparison<br>harmony |
| 6 | halves of the same thing are equal to each other | comparison<br>harmony |
| 8 | the whole is greater than the part | comparison<br>harmony |

We also present the following table of criteria that determine the aesthetic appeal of the triangle and its types:

| shape | property | aesthetic criteria |
|---|---|---|
| triangle | Opposite the larger angle is the larger side and vice versa | comparison harmony |
| isosceles triangle | Opposite two equal sides are equal angles and vice versa | comparison symmetry harmony |
| equilateral triangle | Opposite all equal sides lie equal angles and vice versa | symmetry comparison harmony |
| Golden Triangle | Two sides are equal, and with the third side they are divided in the golden ratio | comparison golden ratio symmetry harmony |

The table shows that an arbitrary triangle, in contrast to an arbitrary quadrilateral, is beautiful because it satisfies the aesthetic criterion of harmony. Note that a quadrilateral may not have such harmony. An equilateral triangle is more beautiful because harmony is more clearly manifested in it: equal angles lie opposite equal sides and vice versa. In addition, this triangle also has one axial symmetry. An equilateral triangle is already much more beautiful, because in it the harmony mentioned above arises not for two, but for all sides. Moreover, the number of symmetries is not two, but six. The Golden Triangle becomes attractive in other ways as well. Unlike an equilateral triangle, it has few symmetries, only two. But its unequal sides are correlated by the golden ratio, which gives the image greater aesthetic appeal, hence its name - the golden triangle.

Thus, the formative criteria of scientific beauty are widely involved in shaping the content of both the Elements and the high school geometry course. However, this reality is not fully taken into account in the learning process. From the point of view of identifying truth, only symmetry is used in some cases to establish certain facts. Although all formative criteria have enormous aesthetic potential, which is present both in the axioms of Euclid’s geometry and in all its materials.

In the case of algebra, symmetry, comparison and harmony manifest through relationships and connections between algebraic operations and equalities and inequalities, given in the form of axioms and properties that follow from them. Here the question of discovering truth both in a linearly ordered field of real numbers and in a partially ordered field of rational fractions is quite reasonable, if we do not take into account the problem of continuity of real numbers. The aesthetic motives of the characteristics also appear here, but in the educational process, they receive almost no attention.

**4. Manifestations of unifying sings of scientific beauty in meaningful axiomatic theories**

The unity of diversity, community, mathematical notation of scientific laws and other unifying signs of scientific beauty play an important role in identifying both truth and beauty in meaningful axiomatic theories. It should be noted that the first two of these signs - unity of diversity and community - were introduced into circulation by F. Hutcheson (Francis Hutcheson, 2004), and the third - by M. V. Volkenshtein (M. V. Volkenshtein, 1931).

The unity of diversity first of all ensures the creation of such an element of scientific truth as a concept. Without this attribute, it is impossible to form concepts - to characterize a group of objects with one and the same attribute using one term. The concepts of meaningful axiomatic theory are also derived: point, segment, triangle and other figures in Euclidean geometry, number, variable, algebraic expression, polynomial, function and other concepts of school algebra.

The same applies to the properties of a meaningful axiomatic theory. And first of all we are talking about axioms. The axioms of equality, addition and multiplication, presented as laws, are a summary of students' knowledge of numbers studied in previous grades, which deepens students' knowledge of truth. They are manifestations of the sign of unity in the diversity of scientific beauty.

The same applies to theorems and their proofs. The Pythagorean Theorem, for example, applies to all right triangles. Moreover, it, or rather its converse theorem, unites all right triangles.

Community plays an equally important role. The general word said about a girl applies to all girls, what is said about the number 5 applies to the elements of a group of 5 objects, what is said about the variable x applies to all elements of its range of values, what is said in the Pythagorean theorem – to all right triangles, etc.

The mathematical notation of a scientific pattern is also an important way of discovering the truth. The formula $S = VT$, for example, allows us to observe the dependence of the distance traveled by a uniformly moving body on the time spent, which is the key to solving one of the most important groups of problems in physics or identifying the truth. These are also the mathematical formulas used in physics, chemistry and other sciences. This is so important that it is now generally accepted that it is impossible to carry out serious research in the natural sciences without the use of these mathematical formulas.

In axiomatic theory, the mathematical record of a scientific law takes on a systematic and perfect form. For this reason, the role of the sign of scientific beauty in identifying both truth and beauty becomes especially important here. At the same time, one can think that Hutcheson considered the community and unity of diversity to be a criterion of scientific beauty precisely because of their connection with the problem of identifying the truth: a sign is beautiful because it makes it possible to reveal the truth.

The same applies to the mathematical deation of a scientific pattern. For example, the beauty of the formulas $(x + y)2 = x2 + 2xy + y2$ and $S = VT$ have different roots. The first is based on symmetry, and the second is a mathematical representation of the laws of physics. It should be noted that Newton liked the sign of a mathematical recording of a scientific pattern so much that he called it proof of the existence of God, because only God could create such “divine” beauty.

Let us imagine the manifestations of the unifying criteria of scientific beauty in the topic “Equality of Expressions” in a high school algebra course H.S. Mikaelian, 2023.

| Criterion | The object in which the trait appears | |
|---|---|---|
| Unity and community of diversity | Equality<br>Law of reflexivity of equality<br>Law of symmetry of equality<br>Law of Transitivity of Equality<br>Equality of expressions equal to the same expression<br>True and false formulas<br>Right and wrong actions<br>Identically equal expressions<br>Identity transformations<br>Identities | |
| Criterion | Scientific Law Mathematical Notation | Scientific Law Mathematical Notation |
| Mathematical notation of scientific law Equality | Equality<br>Law of reflexivity of equality<br>Law of symmetry of equality<br>Law of Transitivity of Equality<br>Equality of expressions equal to the same expression | $x = y$<br>$x = x$<br>if $x = y$ then $y = x$<br>if $x = y$ and $y =z$, then $y = x$ |

| | | |
|---|---|---|
| | | if x = z and y =z, then x = y |

Thus, the unifying signs of scientific beauty are aimed primarily at identifying the truth, and aesthetic attractiveness here should be understood as a process aimed at identifying the truth.

## 5. Manifestations of logical signs of scientific beauty in meaningful axiomatic theories

In meaningful axiomatic theories, the role of the signs of scientific beauty is especially important: logical rigor, clarity, simplicity and reduction of the complex to the simple. Logical rigor is the main feature of axiomatic theory. It is the presentation of axiomatic theories in mathematics and, in particular, meaningful axiomatic theories in school mathematics that most closely correspond to logical laws and patterns. And the main trend here is logical rigor, which is realized through proof. In formal axiomatic theory, proof also receives its clear definition, which, although not clearly manifested in meaningful axiomatic tenors, still retains its essence, the main feature of which is logical rigor. That is, proof is the most important and main means of identifying and confirming the truth in axiomatic theories of mathematics. Every statement of an axiomatic theory is true.

American mathematician and educator Paul Lockhart also calls proof the art of mathematics (Paul Lockhart). Why? If logical criteria, logic are the aesthetics of each theory, then proof is the art of this theory, in which the proof of a mathematical axiomatic theory has reached unprecedented epistemological and aesthetic heights. It should be noted that it is logical rigor, the derivation of judgments from each other, and proof that forms and develops the student's logical thinking, and also influences the formation of a high level of his speech culture.

From this point of view, a certain discrepancy between the sections of algebra and geometry in the standard subject of mathematics of secondary schools of the Republic of Armenia is noticeable (Mathematics Subject Standard (Grades 1-12), (2023),). If the geometry material in the standards is accompanied mainly by the requirements of proof, then in algebra the term "proof" is absent. On the one hand, this does not allow the process of developing logical thinking in students to be fully realized. On the other hand, under these conditions it is very difficult to present algebra as a meaningful axiomatic theory, as is done in the case of geometry.

Another feature of a meaningful axiomatic theory is clarity. It includes both the concepts and properties and proofs of theory. In the case of concepts, clarity involves clear, unambiguous definitions of the concepts in question. This is not only an expression of the aesthetic of clarity, but also the epistemological and aesthetic basis of mathematics, its school curriculums and mathematical language: without a clear, unambiguous understanding there is no mathematics (H. S. Mikaelian, 2019).

The aesthetic appeal of clarity and the desire to reveal truth are also evident in mathematical theorems. Clarity here means, first of all, a clear formulation of the necessary conditions or premises and conclusion, and in the case of school mathematics - their presentation accessible to the student. In the latter case, it is very important that the student understands the role of each of the conditions of the theorem in confirming and proving its conclusion. From a technical point of view, it is usually more difficult to understand the proof of a theorem, which must also be clear: it is carried out in a certain sequence of steps that are true inferences and lead to the conclusion of the theorem. It is worth noting that in meaningful axiomatic theories, the incomplete application of the aesthetic attribute of clarity is due to the incomplete application of the elements of logic. And we are not talking here about identifying the truth through mathematical logic A.T. Mkrtchyan (2018).

It should be noted that the aesthetic of clarity of mathematical material appears along with its perception and understanding. Without understanding there can be neither clarity nor aesthetic appeal. Lack of understanding, mechanical reproduction of the material, which is also a consequence of the lack of clarity, not only does not contribute to the expression of the aesthetic appeal of the material, but also leads to a negative aesthetic reaction, repels the student from the material and from mathematics in general. And even if the student's efforts to demonstrate some knowledge at any cost or the teacher's efforts to get the student to reproduce knowledge turn out to be successful, we only have negative consequences: a student who has learned such answers not only easily forgets the knowledge he reproduced, but also gets used to making hasty, superficial actions in life or professional activity without penetrating into the depth of phenomena. Therefore, the teacher is obliged not to seek an answer from the student at any cost, by correcting one or two words, but to force him to understand the essence of the material, based on the aesthetic sign of clarity, and to correct a mistake if one is made.

Simplicity, as a sign of beauty in a mathematics course, was proposed by V. G. Boltyansky (V.G. Boltyansky, 1973).), G. I. Sarantsev (G. I. Sarantsev, 2000), M. S. Yakir (M.S. Yakir, 1989). The sign of reducing the complex to the simple, as a sign of scientific beauty, was proposed by V. M. Volkenshtein [6]. Simplicity, clarity, its presence make every process beauty, and the complex often becomes incomprehensible, and therefore not beautiful. Therefore, bringing the complex to the simple, simplification is a process aimed both at identifying the truth and at forming the beautiful.

Axiomatic theory is an excellent structure for reducing the complex to the simple, which also applies to meaningful axiomatic theory. This is evidenced by a brief description of the above structure of formal axiomatic theory. And if we turn to school mathematics, then the definition of a concept and consideration of examples about it are nothing more than manifestations of the characteristic of bringing the complex to the simple. Teaching the concept of a number in a math class is a great example of bringing the complex to the simple. Here, learning is carried out through the chain natural number - integer number - rational number - real number, each subsequent link of which is much more complex, defined and understood through its previous link.

The aesthetic sign of reducing the complex to the simple is widely manifested in mathematical theorems and their proofs. Each application of a mathematical theorem is already a reduction of the complex to the simple. How to calculate, for example, the volume of such a complex body as a ball? At first glance, this seemingly complex process can be reduced with the help of mathematical tricks to the simple formula $V = \frac{4}{3}\pi R^3$. And the beauty here is not only in the simplicity of the formula's appearance, but also in the possibilities of its application (H.S. Mikaelian, 2019).

Thus, the sign of scientific beauty of bringing the complex to the simple is also an important tool for constructing and teaching a meaningful axiomatic theory, in particular a school course in mathematics, and a means of identifying the truth and constructing the beautiful.

## 6. Conclusions

This study allows us to draw the following conclusions:

a) Formative, unifying and logical groups of objective signs of scientific or mathematical beauty play important but different roles in the formation and identification of truth and aesthetic appeal in meaningful axiomatic theories.

b) Formative criteria - symmetry, comparison, harmony, rhythm, applicability, tend to form and reveal the beauty of the objects of a meaningful axiomatic theory more than its truth.

c) Uniting criteria - unity of diversity, generality, mathematical recording of scientific laws, are aimed at identifying the truth in a meaningful axiomatic theory. And the beauty of objects here is determined by the connection of signs with truth.

d) Logical criteria - logical rigor, clarity, simplicity, reduction of the complex to the simple, are aimed at the formation and disclosure of both the truth and the beauty of the objects of a meaningful axiomatic theory, which they fully realize.

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