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TOWARDS A CRITICAL EPISTEMOLOGY OF MATHEMATICS

HACIA UNA EPISTEMOLOGÍA CRÍTICA DE LAS MATEMÁTICAS

PARA UMA EPISTEMOLOGIA CRÍTICA DA MATEMÁTICA

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ABSTRACT

This essay addresses a critical epistemology of mathematics as an investigation into the epistemic limitations of mathematical thinking. After arguing for the relevance of a critical epistemology of mathematics, I discuss assumptions underlying standard arithmetic and assumptions underlying standard logic as examples for such epistemic limitations of mathematical thinking. Looking into the work of philosophically interested scholars in mathematics education such as Alan Bishop and Ole Skovsmose, I discuss some early insights for a critical epistemology of mathematics. I conclude that these insights can only be the beginning, that we are yet far away from a proper understanding of the epistemic limitations of mathematics, and that more research is needed.

Keywords: mathematics education. philosophy of mathematics. critical theory. epistemology.

RESUMEN

Este ensayo aborda una epistemología crítica de las matemáticas como investigación de las limitaciones epistémicas del pensamiento matemático. Tras argumentar a favor de la relevancia de una epistemología crítica de las matemáticas, discuto los supuestos subyacentes a la aritmética estándar y los supuestos subyacentes a la lógica estándar como ejemplos de tales limitaciones epistémicas del pensamiento matemático. Analizando el trabajo de estudiosos de la educación matemática interesados por la filosofía, como Alan Bishop y Ole Skovsmose, discuto algunas de las primeras ideas para una epistemología crítica de las matemáticas. Concluyo que estas ideas sólo pueden ser el principio, que aún estamos lejos de una comprensión adecuada de las limitaciones epistémicas de las matemáticas y que se necesita más investigación.

Palabras clave: educación matemática. filosofía de las matemáticas. la teoría crítica. epistemología.

RESUMO

Este ensaio aborda uma epistemologia crítica da matemática como uma investigação das limitações epistêmicas do pensamento matemático. Depois de defender a relevância de uma epistemologia crítica da matemática, discuto os pressupostos subjacentes à aritmética padrão e os pressupostos subjacentes à lógica padrão como exemplos de tais limitações epistêmicas do pensamento matemático. Analisando o trabalho de estudiosos da educação matemática interessados em filosofia, como Alan Bishop e Ole Skovsmose, discuto algumas das primeiras ideias de uma epistemologia crítica da matemática. Concluo que essas ideias podem ser apenas o começo, que ainda estamos longe de uma compreensão adequada das limitações epistêmicas da matemática e que são necessárias mais pesquisas.

Palavras-chave: educação matemática. filosofia da matemática. teoria crítica. epistemologia.

Introduction

According to *The Shorter Routledge Encyclopedia of Philosophy*, epistemology "is concerned with the nature, sources and limits of knowledge" (Klein, 2005, p. 224). The philosophy of mathematics seems to be more interested in the nature and sources of mathematical knowledge: See, for example, the classic claim that the *nature* of mathematical knowledge is its certainty or the modern discussion which kind of proof and rigour is acceptable as *sources* for the validity of mathematical knowledge. In contrast, there appears to be little discussion about the *limits* of mathematical knowledge. See, for example, how Gödel's incompleteness theorems destroyed Hilbert's program to prove the consistency of mathematical theories, or rather the silence and ignorance which followed on this event in the philosophy of mathematics (Hersh, 1979).

All these examples and much of the research in the philosophy of mathematics are concerned with issues internal to mathematics and not with issues of applying mathematics. This is a problem, for, as Wittgenstein (1978) put it, "it is the use outside mathematics […] that makes the sign-game into mathematics" (p. 257); or, in other words, the usefulness of mathematics is its *raison d'être* as an academic discipline. We may separate two different domains of knowledge, which could both be called "mathematical": There is knowledge about mathematical concepts, structures and methods; and then there is knowledge which builds on the use of such structures, for example in physics or when prescribing an income tax. Somewhat misleading, these domains are often called "pure" and "applied" mathematics (Hacking, 2014). (Considering Wittgenstein's thought above, "pure" mathematics cannot be all that pure. *Vice versa*, Maddy, 2008, argues that "applied" mathematics is often more theory- than empirically driven.) Despite Galilei's (1610) popularised notion that the book of nature is written in the language of mathematics, there is clear evidence that a mathematical view on the phenomena of our world directs and limits our understanding of these phenomena, even in physics (Cartwright, 1983; Lindley, 2020). As a side note, see how little the research field on modelling in mathematics education is concerned with this issue (Kollosche, 2021, p. 482). In mathematics education research, Skovsmose (1994) coined the notion of the "formatting power" of mathematics (p. 43): Mathematics influences how we think, how we perceive our world, and how we mould it. Ever since, a part of Skovsmose's work was concerned with explaining the ways in which mathematics formats our reality. I will return to the ideas of him and others in the next section of this paper.

Quite generally, we might raise the question why we, especially as researchers in mathematics education, should even care about epistemic limitations of mathematics. I argue that an understanding of the mechanisms in which mathematics offers, directs, beguiles, represses and displaces ways of understanding our world

1. should be an integral part of any mathematical literacy in order to be able to act as a self-determined and critically informed citizen (Skovsmose, 1985),
2. assists us in understanding cognitive obstacles in learning mathematics (Schneider, 2020),
3. allows us to understand the interest in or the rejection of mathematics (Kollosche, 2018),
4. enables us to identify epistemological biases that discriminate specific social groups (e.g, Mendick, 2006)

Given that these dimensions feature critical perspectives on mathematics, on learning mathematics, and on identifying with mathematics, and given that we now put a stronger emphasis on the epistemic *limits* of mathematics, I will talk about a *critical* epistemology of mathematics. One might think that developing such a critical epistemology of mathematics should be a task for the philosophy of mathematics, but, as Dörfler (2003) argued, the close proximity of mathematics education research to the meta-study of mathematics and the lack of a discipline that would feel responsible for this meta-study entail that it often has to take place within mathematics education research itself.

The relevance of a critical epistemology of mathematics can be seen at least in three domains: First, research shows that part of the popular aversion against mathematics is based on a critique of its epistemic premises (Kollosche, 2019). Such critique has already been addressed in philosophy (e.g., Horkheimer & Adorno, 1944/1997) and in mathematics education research (Skovsmose, 2021), yet on the basis of a fragmented theory of a critical epistemology of mathematics. Second, the last decades have seen an increasing awareness for unwanted consequences of the use of mathematics (e.g., O'Neil, 2016; Porter, 1996; Skovsmose, 2005), but these discussions have to remain limited to the consequences alone as long as there is no robust theory of the epistemic limits of mathematics which could explain such consequences. Third, despite the fragmentation of the critical epistemology of mathematics, proponents, especially within mathematics education research, have proposed a reorientation of the epistemology of mathematics. I immediately recall Chapman's (1993) and Burton's (1995) call for a "feminist epistemology of mathematics", but there might be more contemporary examples such as Gutiérrez' (2012) attempt to reconcile mathematics with indigenous epistemologies. However, what these attempts lack is a reliable account of the epistemological limits of mainstream mathematics, against which such reorientations could be developed.

Two short examples

I want to give two short examples to make my theoretical remarks about epistemological limits of mathematics more concrete. Elsewhere, I discussed how the equation 2+2=4 is taken as an absolute mathematical truth by some, while others try to find examples where 2+2=4 does not hold true (Kollosche, in press). On the one hand, there is no doubt that 2+2=4 is a true statement that can be deduced from any axiom system of the arithmetic of natural numbers. On the other hand, there are limitations to the application of the arithmetic of natural numbers; for example, that any countable object stays one and one object (and does not merge with others, does not destroy others, does not create new objects as offspring, does not itself become less than one). *Vice versa*, this means that, applying the arithmetic of natural numbers on a phenomenon, we presume – consciously or not, justified or not – that this phenomenon has the required properties. This is how mathematics can format our perspective on reality.

The first example addresses only one limitation of one mathematical theory (although a central and elementary one). The more pressing question for the epistemology of mathematics is whether there are epistemological limitations that affect mathematics as a whole (or at least most of it). This task can quickly lead to the question what mathematics is – in an attempt to describe its epistemic nature and derive its limitations from that description. However, as we lack a consensus on the definition of mathematics (Hacking, 2014), it might be a pragmatic step to pick epistemic qualities that we agree mathematics to have, and to start from there. For example, even though there is a niche for experimental mathematics and for non-standard logics, mathematics is usually expected to prove its statements on the basis of a logic whose basics have been developed in Ancient Greece. This includes the laws of the excluded middle and of the excluded contradiction, which, taken together, hold that any statement is either true or not true (Kollosche, 2013). But not all worldly statements subject to such a dichotomy: For some people, "Are you in a relationship?" is not a question that can easily be answered with "Yes" or "No". The same might hold true for the question "Are you a woman?". Note that a dating website might actually ask these questions and use the answers for a mathematical model to facilitate romantic relationships or sexual encounters. The last question also shows that the problem is not that the question is underdefined: We could ask more specifically "Is your biological sex female?", but medicine teaches us that even this question cannot in all cases simply be answered with "Yes" or "No". Thus, again, mathematics, for example, that of our dating website, requires very specific epistemic presumptions in order to be applicable. And vice versa, wherever mathematics is applied, such epistemic presumptions have been made – consciously or not, justified or not. The interesting epistemological question then is: What are these epistemic presumptions of using mathematics? And, returning to my initial question: What are the epistemic limits of mathematics?

Some first answers

I have not yet found any comprehensive theory of the epistemic limits of mathematics, and I am afraid that there is none yet. However, there are answers scattered throughout the research literature in mathematics education and beyond. In this section, I will present and discuss some of these.

Bishop's values of mathematical culture

In his seminal book *Mathematical Enculturation*, Bishop (1988) claimed that "*educating* people mathematically […] requires a fundamental awareness of the *values* which underlie mathematics" (p. 3, original emphasis). He devoted a chapter of his book to the discussion of "the values of mathematical culture" and lists the following aspects:

1. Bishop claimed that *rationalism*, for him defined by "its focus on deductive reasoning as the only true way of achieving explanations and conclusions", is "an ideological component of culture" and "at the heart of Mathematics" (p. 62). Given Bishop's study of non-Western mathematics, which includes mathematical traditions that did without deduction, it is surprising how uncritical Bishop was about this ideology. Concerning the verb "rationalise", he observed that it "seems to have developed some negative connotations" and found it "difficult to see clearly why this should be" (p. 63). Instead, the praised the "beauty of completeness and wholeness about logical argument, where the 'loose ends are tied up', where 'fuzziness' and imprecision are replaced by clarity and certainty, where greyness and shadowy half-truths are illuminated by the bright light of reason" (p. 64). However, if rationalism is an epistemic presumption of mathematics, then we should also ask for the epistemic limitations it induces.
2. Bishop used the term *objectism* to "characterise a world-view dominated by images of material objects" as opposed to a world-view "based on 'processes'" (p. 65). He refers to the Pythagorean tradition to understand the world through its objects and not through its changes and flows. Bishop linked this thought to the observation that "the Westerner" strives to describe objects independent from their observers, while other cultures deemed that impossible and focused on our human and sometimes even subjective relations to these objects (*ibid.*). Mathematics then turns on itself by objectifying its abstractions "which enables them to be handled so precisely" (p. 66). Limitations came in sight when Bishop stated that rationalism and objectism are ideologies which are "in some sense dehumanised", "divorced from their human creators" or "based on inanimate objects and not on animate phenomena" respectively (*ibid.*). Later, he added that "materialism developed a picture of reality as some kind of complex mechanism, with nature being composed of objects moving in ways akin to machinery", which "has, as we know, brought its benefits and also its drawbacks" (p. 68).
3. Bishop described *control* as a "sentiment" of mathematics. For him, "knowledge is about control" in the sense of being able to predict (p. 70). He underlined that "Mathematics, through science, is again being used to further our control over the environment […], and the idea of

Mathematics as a *tool* for gaining control is once again being strongly reinforced", also in the social realm (*ibid*., original emphasis). Indeed, already before any application, "Mathematics so clearly is about control", for "facts are facts, theorems are proved", that is, "the abstract objects under consideration behave predictably, [*sic*] and according to the well-formulated rules of the Mathematical game" (p. 71). Bishop acknowledged that "control, however, is a double-edged sword since, in order to control something, one's behaviour also needs to be modified" (*ibid.*), yet his following thoughts are more concerned with the technological live-world we are thus trapped in and not with the epistemic perspectives we have thus subjected us to.

4. Bishop listed *progress* as another "sentiment" of mathematics (p. 72). However, although Bishop explained how there is progress in and through mathematics, he did not argue that progress is a sentiment that distinguishes mathematics from other epistemic perspectives. Therefore, I will skip this value in my discussion.
5. The next "value" that Bishop mentioned is *openness*, meaning that "Mathematical truths, propositions and ideas generally, [*sic*] are open to examination by all" (p. 75). What Bishop means is that mathematical truths come with an account of why they should hold true, and that the interested critic of such truths needs nothing but a sound mind to investigate them. Eventually, "you can convince *yourself* that any Mathematical principle is true, nobody has to persuade you" (p. 76, original emphasis). This appears to be more of an in-principle argument, as not many people dive into mathematical theories without having a fitting vocational background, and as access to mathematics is not as equally distributed as these philosophical considerations might suggest (Bishop et al., 2015). It is also an argument that is not limited to mathematics but again applies to all academic endeavours.
6. The last "value" that Bishop (1988) addressed was *mystery* (p. 77). He explained that much of mathematics was not understood by laypersons, maybe not even by most mathematicians, and that mathematicians are mysterious themselves. While the last claim is discussed on a polemic basis and problematic in my point of view, also the rest is not very enlightening, as it might be said about any highly specialised field of academic practice. Maybe Bishop hinted into an interesting direction when he addressed that "mystery in Mathematics is related to the act that one is dealing with abstractions", adding that "the more abstract the ideas become, the less contextualised they will be and therefore the less meaningful also" (p. 80). However, there is no critical reflection of this special trait either.

All in all, Bishop's (1988) explanations are sometimes too general and do not address mathematics specifically; and if they do, they are not discussed critically. The literature sources for his discussions are not very broad and the discussions stay short. To his defence, we have to admit that it was never Bishop's intention to write a critical epistemology of mathematics.

Skovsmose's limitations of the subject

The "limitations of the subject" have been in the focus of Skovsmose's work since the very beginning of his academic work (Skovsmose, 1985, p. 341). Although he has never attempted to provide a comprehensive theory of the epistemic limits of mathematics, he has discussed such limits on several occasions. Here, I will restrict myself to discussing some ideas from his books *Travelling Through Education* (Skovsmose, 2005) and *In Doubt* (Skovsmose, 2009).

In *Travelling Through Education*, Skovsmose (2005) built on Herbert Marcuse when he addressed *instrumental reason* as "a way of thinking which […] is linked to natural science, and which treats the objects of scientific investigation as simply *objects*" (p. 108, original emphasis). He reported that "Marcuse links mathematics with one-dimensionality" and that mathematics "establishes the grammatical foundation for a projection into a one-dimensional universe, by means of which the 'immediate experiences' become stripped of 'judgement'" (p. 109). This way of thinking is considered problematic when "imported by social science", where it "gains an illegitimate power of suppression and social manipulation" (p. 108). Here, we see a theoretically more robust introduction to what Bishop (1988) called an ideology of objectism. In *In Doubt*, Skovsmose (2009) wrote about the ideology of a "mechanical world", which was developed in early Modernity by Galilei and Descartes, and which thinks "of the world as operating like a machine, like clockwork, where all cogwheels [or objects, we may add] are connected and interacting" (p. 45). Here, Skovsmose succeeded to present an interesting twist to the idea of objectism, as not all phenomena, for example not gravity, fit easily into a mechanical world view, and as it then is the transition to a *mathematical-mechanical world view*, which allows to incorporate such phenomena into the theory of nature while preserving the primacy of its mechanical assumptions (*ibid*). Bringing Bishop's (1988) idea that mathematics is objectifying its abstractions back into play, we may propose that the benefit of mathematics here is that is has no problems to objectify abstract ideas such as gravity, while natural science with a mechanical world view alone might have found this impossible. Thus, we arrive at the idea that mathematics and natural science might be in a state of interdependence when it comes to objectism. However, why and how this is so, and if it is necessarily so, remains unclear in both sources.

A new consideration about the epistemic limits of mathematics can be derived from Skovsmose's (2009) discussion of Frege's distinction between sense (*Sinn*) and reference (*Bedeutung*) (p. 46). Here, the sense of a concept such as "triangle" would be the full range of associations that come with this concept, while its reference would be its unambiguous logical identification, given for example by a definition or by naming the set of elements that fall under this concept. Statements about these concepts can be 'made sense of' from many different perspectives, but they have only one reference, namely their truth value (true or false). Frege's service to the philosophy of mathematics was to point out that Modern mathematics is concerned with reference alone and must ignore questions of sense. This distinction can explain and justify the effectiveness of abstract mathematics, but obviously it cannot explain how mathematics is applied. Nevertheless, as Skovsmose pointed out, later Modernity saw the rise of a *formal world view*, where logic and mathematics have the task to study "all possible valid forms of reasoning" (p. 49), while any applications of such forms lie beyond the mathematical endeavour. If we subscribe to this view, then we face two different questions: First, what would be the epistemic limits of pure mathematical reasoning, and second, what would be the epistemic limits of applying mathematics? However, if we side with Wittgenstein's (1978) notion that "it is the use outside mathematics […] that makes the sign-game into mathematics", implying that the epistemology of "pure" and "applied" mathematics cannot be understood separately, then we see that a formalistic view on mathematics has concealed this connection and may have hindered the analysis of epistemic limits of mathematics. For an example, just revisit the end of our discussion of objectism in the paragraph above: While the epistemic limits of objectism become all too obvious in applications of mathematics, it remains unclear if mathematics is inseparably bound to objectism, it if would be possible to do non-objectifying mathematics, or if mathematics already is a means to overcome objectism.

Conclusion and outlook

As the scattered answers above may exemplify, the idea of a critical epistemology of mathematics describes a complex and highly relevant field of inquiry. But these answers also show how random it yet seems who discusses which issues in the field on the basis on what references. Concerning the references, just note that Skovsmose does not even reference or discuss Bishop's earlier ideas in this context. Concerning the randomness of the issues discussed, just consider the following (incomplete) list of additional questions that would fall under a critical epistemology of mathematics:

- Under which conditions (or for which price) can phenomena be counted and measured?
- Under which conditions (or for which price) do theories subject to the logic of either-or dichotomies and deduction?

- Under which conditions (or for which price) can practical questions be answered on the basis of a formalistic algorithm?
- Under which conditions (or for which price) can a concept become a concept in a mathematical theory?

How may we proceed? Apart from the fact that any beginning of a critical epistemology of mathematics deserves a more profound and comprehensive study of what we already know in mathematics education research than I can provide within the limits of this paper, and apart from the fact that a critical epistemology would benefit from establishing a line of discussion where relevant publications refer to each other, it may be fruitful to look for inspiration beyond the limits of mathematics education research, maybe in in the philosophy of mathematics, but certainly in general philosophy. For the latter, I want to provide short teasers to three promising sources:

- Although Horkheimer and Adorno's (1944/1997) *Dialectic of Enlightenment* includes only short comments about the epistemic limits of mathematics and does not provide references to the sources of these comments, it has been a welcome source for the critical investigation of the epistemology of mathematics, also for some of the studies cited here (e.g., Porter, 1996; Skovsmose, 2009). By the way, note that Belenky et al. (1986) seem to rely heavily on the *Dialectic of Enlightenment* in their book *Women's Ways of Knowing*, which also appears interesting for our cause. A step forward in departing from the *Dialectic of Enlightenment* would be to systematically analyse its comments about mathematics and epistemology, and to trace these comments back to earlier philosophers.
- While Hegel built his philosophy on such basic concepts as identity and negation, which might be very close to our standard understanding of mathematics, Deleuze (1968/1994) started an attempt to replace these by the basic concepts of difference and repetition, which gives rise to a very different philosophy and has the potential to understand mathematics as a discipline that is built on such basic epistemic practices as repeating an action and marking a difference. I am grateful to Liz de Freitas for pointing my attention in this direction (and I hope that one day I will also have the time to fully explore this direction).
- It is due to Felix Lensing that I can also list the work of Husserl, a world-rank philosopher who goes strangely unnoticed by mathematics education research and the philosophy of mathematics, although he had a doctorate in mathematics and published extensively about philosophy and mathematics. In his early work, Husserl (1891/2003) developed an introduction to the natural numbers on the basis of descriptive psychology, thus identifying specific thought processes that precede the idea of the natural numbers and are inherent to the use of arithmetic. It might be able to reinterpret these thought processes as epistemic assumptions of arithmetic, and they could thus demarcate epistemic limits of mathematics.

Exploring these sources will allow to widen the theoretical background of a critical epistemology of mathematics, but will not provide the field with any structure. Therefore, a next step may be to elaborate further on epistemological limits in general, to devise a possible logic or structure for their study, only to then apply it to mathematics.

I hope that my theoretical considerations, my practical examples, and my insights into selected studies, have at least convinced the reader that such an endeavour is important, if not even motivated the reader to contribute to it.

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