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ETHICS AS PART OF MATHEMATICAL REASONING IN SHARING

A ÉTICA COMO PARTE DO RACIOCÍNIO MATEMÁTICO NO COMPARTILHAMENTO

LA ÉTICA COMO PARTE DEL RAZONAMIENTO MATEMÁTICO EN EL COMPARTIR

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RESUMO

Há uma maior necessidade na sociedade de hoje, para entender e discutir criticamente como os recursos limitados do nosso planeta são alocados. Freqüentemente, modelos matemáticos são usados em relação a problemas de alocação de recursos, e uma visão comum é que a matemática em si é neutra. Neste artigo, desafiamos essa visão da matemática como uma prática neutra por meio de uma análise de possíveis soluções para uma tarefa de compartilhamento. As tarefas vêm de um projeto de pesquisa com o objetivo de estudar como a matemática pode apoiar o raciocínio ético e os argumentos éticos podem apoiar diferentes soluções matemáticas ao compartilhar um recurso. No raciocínio ético, três componentes são abordados: Informação, Coerência e Engajamento. Mostramos que o raciocínio ético faz parte do raciocínio matemático em todas as soluções da tarefa, independentemente de o dividendo ser tratado como indivisível ou divisível.

Palavras-chave: ética. raciocínio matemático. compartilhamento.

ABSTRACT

There is a greater need in today‘s society, to understand and critically discuss how the limited resources of our planet are allocated. Often, mathematical models are used in connection with resource allocation problems, and a common view is that mathematics in itself is neutral. In this article, we challenge this view of mathematics as a neutral practice through an analysis of possible solutions to a sharing task. The tasks come from a research project aiming to study how mathematics can support ethical reasoning and ethical arguments can support different mathematical solutions when sharing a resource. In ethical reasoning, three components are addressed: Information, Coherence, and Engagement. We show that ethical reasoning is part of mathematical reasoning in all the solutions to the task, independent of whether the dividend is treated as indivisible or divisible.

Keywords: ethics. mathematical reasoning. sharing.

RESUMEN

Existe una mayor necesidad en la sociedad actual de comprender y discutir críticamente cómo se asignan los recursos limitados de nuestro planeta. A menudo, los modelos matemáticos se utilizan en relación con los problemas de asignación de recursos y una opinión común es que las matemáticas en sí mismas son neutrales. En este artículo, desafiamos esta visión de las matemáticas como una práctica neutral a través de un análisis de posibles soluciones a una tarea compartida. Las tareas provienen de un proyecto de investigación que tiene como objetivo estudiar cómo las matemáticas pueden respaldar el razonamiento ético y los argumentos éticos pueden respaldar diferentes soluciones matemáticas al compartir un recurso. En el razonamiento ético se abordan tres componentes: Información, Coherencia y Compromiso. Mostramos que el razonamiento ético es parte del razonamiento matemático en todas las soluciones a la tarea, independientemente de si el dividendo se trata como indivisible o divisible.

Palabras clave: ética. razonamiento matemático. compartir.

Introduction

On this planet there are limited resources: the Earth Overshoot Day – the day that marks when humanity has exhausted nature's budget for the whole year – in year 2021 was July 29th and most countries surpass their own limits already in the first six months of the year (Lin et al., 2021). Not only are resources sparse and that some countries use more resources than others, if you add that resources should be allocated in a way so that agents' preferences are taking into account for, then you have, according to Suksompong (2021), one of the fundamental problems in society. In her groundbreaking book, Nicholas (2021) shows how if we want humans to take action for climate change, where every fraction of degree matters for this planet, information – facts and science results – is not enough. Researchers focusing on sustainability commitment agree, and points out the importance of affective aspects such as values, emotions, and motivation as part of decision making (e.g., Öhman & Sund, 2021). Such reasoning is a social process, where negotiation and critical thinking often are part of it (Vare & Scott, 2007). The process is also referred to as moral reasoning (Samuelsson & Lindström, 2020) or ethical reasoning (Sternberg, 2012; Sumpter & Hedefalk, 2023). The aim of such reasoning is to answer the question "How can we humans live well in the world?", a question with no easy answers (Griffiths & Murray, 2017). Nicholas (2021) calls it a game of Jenga that we cannot afford to lose. In these complex situations, where there is not easy to know what is the right or wrong decision, mathematics is often used as a tool (Birhane & Sumpter, 2022; Ernest, 2020). One example is how one life is valued and through mathematical reasoning and modelling is given a monetary value (Skovsmose, 2020). At the same time, mathematics is often taught and viewed as neutral and free of values (Ernest, 2020), meaning that ethics is seen as something that is disjoint from mathematics. Here, we would like to challenge this view, by analysing possible solutions to a mathematical task. We align with Buell and Piercy (2022) when they conclude that "one must go beyond the implications or applications, and focus on an "ethical consciousness" (Buell & Piercy, 2022, p.4). Hence, we need to illuminate ethics in mathematical solutions. Buell and Piercy (2022) also refer to Anna Alexandrova's statement from 2018 that no research – no matter how pure one might think it is – is free from social responsibility.

Previous researchers have discussed ethics and mathematics, for instance Skovsmose (2020) and Ernest (2020). They offer a discussion on a macro-level. Here, the aim is to contribute to the "ethical consciousness" (e.g., Buell & Piercy, 2022) by study how ethics can be part of mathematical reasoning on a micro-level. The research question is: How and in what ways can different mathematical reasoning be dependent on ethical arguments in a sharing task?

Background

Starting with ethical reasoning, there are several frameworks and models to describe ethical reasoning (e.g. Samuelsson & Lindström, 2020). Some studies have a starting point that the individuals need to know about certain ethics before one can engage in ethical reasoning. One example of such study is Tväråna (2018) who recommends three types of ethics when discussing sharing with young children. This means that the children/ students first learn about ethics, the apply it on different situations. The other point of view is represented by researchers saying that instead of starting with different theory of ethics, one start with the problem at hand and see what type of ethical reasoning that emerge (e.g. Samuelsson, 2020). Such approach is more in line with the idea of the need for critical thinking as part of decision making (Vare & Scott, 2007). It means that in the different reasoning, different ethics can be used such as ethics of care, consequentialism, virtue ethics and so forth. (Sumpter & Hedefalk, 2023). One framework that allows such flexibility is Samuelsson (2020) that uses three components: information (I), coherence (C), and vividness ('livaktighet', L). In a series of empirical studies, we tested this framework on young children (e.g., Eriksson et al., accepted; Hedefalk et al., 2022). In these studies, we struggled to see how and in what way an argument could fulfil the last component, vividness given it is defined that one need to as vivid as possible present one's argument so that other people can understand your standpoint and you understand theirs (e.g., Samuelsson, 2020). Thus, it required rather elaborated arguments. This could be the result of that the children in our study were young whereas Samuelsson's (2020) framework was developed using data from older student. It is plausible to think that ethical reasoning can differ with age. However, given that young children also should be able to develop their critical thinking and ethical reasoning, the theoretical framing should still be able to capture their reasoning even though it might not be as advanced as older student. After some further reading, an adjusted version was developed. In the adjusted version, presented in Hedefalk and Sumpter (forthcoming), we suggest that instead of looking at the vividness of the argument, the third component should be Engagement (E). The decision stem from how other researchers stress the need to understand affect in ethical reasoning and how affect is entangled with the social context (e.g., Öhman & Sund, 2021; Nicholas, 2021). Therefore, Engagement covers social aspects such as making decisions with others or the context in mind, cognitive aspects such as an understanding that decisions can have consequences, and affective aspects such as expressing a feeling that it is important to act. An example of a sharing situation where values are expressed with respect to the context is Stemn (2017). Students in Liberia were asked to share $45 between three children of different ages:

> One student said that they decided the oldest child would receive $20, the middle child would receive $15, and the youngest would receive $10. This method of sharing money and other items is not uncommon in many African cultures (Stemn, 2017, p. 391)

The students in Stemn's (2017) study had good argument to why one should share the money in such a way, and the study describes how values and ethics are embedded in a culture. It also exemplifies how these values, including ethics, have an impact on what is considered a reasonable mathematical solution, as in the quote above. The solution {20,15,10}is both Informed, Coherent, and the students had arguments that indicated Engagement. In our research, we want to use the framework on young children that might not be as verbal as older students, and therefore the decision is to view an argument as a sign of Engagement if it fulfils one aspect or more, but not necessarily all three aspects (Hedefalk & Sumpter, forthcoming). In the arguments, different types of ethics can be used, although we align us with Samuelsson (2020) and state that no specific ethics need to be introduced in beforehand (Sumpter & Hedefalk, 2023).

Continuing with mathematical reasoning, it is here seen as a social process (e.g., Sumpter & Hedefalk, 2015; Sumpter, 2016). Although this is a theoretical paper, we will use the same theoretical framing as in the empirical papers since, we argue, that the focus is on the content of the arguments, not the process of creating them. The theoretical framing builds on research in mathematical reasoning (e.g. Eriksson & Sumpter, 2021; Lithner, 2008; Sumpter, 2016). In Sumpter (2016), the relationship between mathematical reasoning and argumentation is briefly discussed using Toulmin's (2003) work on order to talk about the role of different argument. It is seen as the process of convincing someone for a specific step in the reasoning. Mathematical reasoning is then the process from meeting a (sub-)task to possibly reaching some conclusions. The reasoning structure uses fours steps (Lithner, 2008): (1) Task situation

(TS); (2) Strategy Choice (SC); (3) Strategy Implementation (SI); and, (4) Conclusion (C). It is important to note that this is theoretical structure and human activity does not follow such a linear description. For each step, different arguments can be expressed: Identifying, Predictive, Verifying, and Evaluative (Lithner, 2008; Eriksson & Sumpter, 2021). The arguments can be anchored in different components that have mathematical properties. These properties are objects, transformations, and concepts (Lithner, 2008). Depending of the task, the mathematical properties might be more or less relevant to the Task Situation. Here, the focus in on sharing and division, two concepts that have overlapping mathematical properties. The main difference is that sharing can accepts unequal shares whereas division means equal partitioning (Correra et al., 1998). However, both transformations means that one need to know the amount that should be shared (dividend) and some attention to the recipients (divisor), where another word for recipients is agent (Aziz et al., 2022). From a mathematical perspective, we also need to define if the dividend is divisible or not. The first situation allows results that includes rational numbers whereas the latter, indivisible, means that solutions are using natural numbers (including zero).

Methods

The task comes from a research project aiming to study young children' (age 3-8 years old) mathematical and ethical reasoning, where six cases were designed (Sumpter & Hedefalk, 2023). The six cases describe sharing scenarios where biscuits should be allocated to soft toys within different contexts, all spanning from no information at all (neutral) to situations where different needs are expressed. The selected task is that four biscuits, of the same size, should be shared amongst three soft toys, with no further information about the soft toys. Sharing is a good topic since it covers both mathematical properties and ethical dimensions, and the concept 'fair share' does not have one unique definition (Hedefalk et al., 2022). As stated earlier, if the solutions are to weigh agents' preferences, the complexity increases (Suksompong, 2021). Here, we do not include dimensions such as the child being one of the agents or that the items can be viewed positive, neutral, or negative (e.g., Aziz et al., 2022). As stated earlier, the project is framed within a theoretical framework where the focus is on the content of different argument, see Figure 1 (Hedefalk & Sumpter, forthcoming):

Argumentation →

Reasoning ↓

Reasoning structure	Arguments	Ethical reasoning
Task Situation	Identifying arguments	Informed Coherent Engagement
Strategy Choice	Predictive arguments	Informed Coherent Engagement
Strategy Implementation	Verifying arguments	Informed Coherent Engagement
Conclusion	Evaluative arguments	Informed Coherent Engagement

Figure 1: Overview of steps in reasoning and different arguments.

The starting point is that no specific ethics is needed in beforehand, and the tasks were also designed with this in mind (Sumpter & Hedefalk, 2023). The data comes from first solving the task in different ways where both authors separately aimed to provide as many different solutions as possible. Each solution was then compared to previous ones to ensure that the mathematical reasoning differ from each solution. As a second step, the solutions were structured using the four steps of reasoning. Possible arguments, mathematical and ethical, were identified and analysed where the component of the argument was in focus. The mathematical analysis focused on the different concepts, objects, and transformation of the reasoning (e.g., Lithner, 2008; Sumpter, 2016). The ethical analysis looked at if any Information might be needed in order to do the mathematical reasoning, or where it might be a gap in the Coherence, or possible Engagement in the reasoning (e.g., Hedefalk & Sumpter, forthcoming). The aim here is not to say which ethics that must be part of an argument, but instead to show how different ethical argument can fill possible gaps in the mathematical solutions.

Results

The solutions are divided into three clusters. The first cluster is if one allows the items to be indivisible, the second one is solutions treating the items as divisible, and the third one is more philosophical.

Indivisible

Here, the items are indivisible meaning that we operate with natural numbers. The task is as followed:

> For natural number $s \in \mathbb{N}$, in Case 2 $[s] = \{0,1,2,3,4\}$, where the set of indivisible items $O$are the same as s, and the numbers of agents (recipients) $[N]$ are fixed to 3.

The first possible solution could be {1,1,1} with *r* = 1. However, in the instruction it states that all biscuits should be shared. Hence, a remainder is not allowed. Three possible solutions are then {2,1,1}, {1,2,1}, and {1,1,2} which from a mathematical point of view are the same solutions since the order is just shifted. However, looking at it from an applied mathematical point of view, they are different since the position of the numbers signal which agent who got what. Independent of the order, the Conclusion is that one agent have one more item which raises questions about Strategy Choice and Strategy Implementation (see Figure 1).

The dilemma who will get the extra biscuit can be solved in different ways. The first way is an informed reasoning, where some explicit need or virtue is taken to account and argued for as a fair share. Hence, the solutions ask for sort of normative ethics to be used when interpreting the Task Situation (identifying arguments) and providing argument for the Strategy Choice (predictive arguments). This could be applied for any distribution of the four items. There are three ways of ordering {4,0,0}, three ways of ordering {2,2,0}, three ways of ordering {2,1,1} and six ways of ordering {0,3,1} to give a total of 15 different solutions. Going back to {2,1,1}, {1,2,1}, and {1,1,2}, another option is that the surplus item is allocated using randomness. If a method is found (e.g. die rolling) whereby all agents have the same probability, $p = \frac{1}{3}$, then the expected outcome could be seen as 'fair', given that it is $\frac{4}{3}$ albeit indivisible items. Nevertheless, the use of randomness needs to be accepted by the agents meaning that identifying arguments and predictive arguments have to be provided so the Strategy Choice of using randomness is seen as an opportunity.

Randomness can be applied to any distribution of the items. For example, one of the 15 solutions can be chosen uniformly at random. The expected outcome is, just as with randomness, $\frac{4}{3}$, but the result can be of much greater inequality. As an extreme, one agent can receive 4 items and the other two will then have nothing. One can compare the reasoning to rolling a die where the expected outcome to get a six is $\frac{1}{6}$, $P(X = 6) = \frac{1}{6}$, but since you only have one roll, the actual outcome can differ. In a similar manner, when sharing four biscuits to three agents, the Conclusion will always be different compared to the expected outcome since the items are indivisible. Independent of which Strategy Choice one chooses and argues for, or how it is implemented, the mathematical reasoning will depend on an ethical reasoning aiming to claim that one Strategy Choice is 'more fair' than another. From the receiving agents' point of view, they need to accept the Strategy Choice and the different consequences they entail. Such acceptance is part of Engagement.

Divisible

There are several solutions if one expands to rational numbers. Then the task is interpreted to,

> For rational number $s \in \mathbb{Q}$, in Case 2, $0 \leq [s] \leq 4$, where the set of divisible items are $[O] = \{0,1,2,3,4\}$, and the numbers of agents $[N]$ are fixed to 3.

The first solution is using the transformation division as the Strategy Choice, which means that each agent gets $\frac{4}{3}$ of the items (Conclusion). The ethics involved is that fair share then automatically means that each agent get the same amount independent of different need or virtue. The consequence is that the only possible Strategy Choice is to divide the items into equal sized area/ volume. The transformation 'to measure' is then needed in the Strategy Implementation in order to make sure that each agent get the same amount (volume/area).

Another variant of this is if the biscuits are allocated first one by one, and the remaining is divided into four quarters. Each of the quarters are allocated, one by one, and the remaining bit is the divided into four quarters. This procedure repeats until there is no physical object that can be divided and shared out. Each agent then receives,

$$
\begin{aligned}
& 1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\cdots \\
= \quad & \sum_{i=0}^{\infty}\left(\frac{1}{4}\right)^{i} \\
= \quad & \frac{1}{1-\frac{1}{4}} \\
= \quad & \frac{4}{3}
\end{aligned}
$$

The solution, an infinite series that is a geometric series, has the same Conclusion as above, but different Strategy Choices and Strategy Implementation. It also requires some sort of measurement, however not as complex as when dividing with three since dividing with four can be operationalised as 'half of a half'. It also has some practical issues since it is only in theory one can divide infinite times. The question of when to stop entails that ethics has to be used, since although it is just a tiny crumb left of the biscuit, the task says that all biscuits should be shared. It raises the question "What to do with the final crumb that practically cannot be divided?". Similar reasoning is then needed just as with indivisible items.

Philosophical

The last solution is a more philosophical solution, building on indivisible items. Let us say it is possible to build a quantum computer device that picks the last biscuit and places it in three black boxes, such that whether or not the biscuit is in any particular box is determined by a series of unobserved subatomic events. Using quantum mechanics, we can show that it is impossible to tell whether a biscuit is in a box until one of the boxes is opened. If one of the agents opens their box and finds a biscuit, it is now guaranteed that there is not a biscuit in the other boxes. If the first agent finds no biscuit, the second and third agents still do not know who has the biscuit. This design can be supported with a solution of Schrödinger's equation which proves that the machine works.

Looking at the ethical reasoning, there are a few issues that needs to be agreed upon. Not only is the assignment of biscuits random, but it also allows the agents to have a biscuit and not having a biscuit at the same time, which presents a challenge if you are in a particular need or if there is a situation where ethics of care is explicit. The agents would need to be able to understand and trust Schrödinger's equation in order to have a complete treatment of the problem. Although possible, one might wonder what challenge such strategy choice would entail? It would be a challenge to claim that the reasoning is Informed and has Coherence, even it all agents involved would express positive Engagement. It raises questions such as "What other, unknown solutions might be suggested?", "What else do we need to know to understand how resources should be shared?", and "Can we only have quantum physicists as arbitrators of sharing problems?".

Discussion

The present paper aims to study how ethics can be part of mathematical reasoning, and the research question is "How and in what ways can different mathematical reasoning be dependent on ethical arguments in a sharing task?". The results showed that looking at sharing, here four biscuits shared by three agents, had several solutions that all entailed different types of ethics. Mathematics is not ethics-free (e.g., Ernest, 2020) and the different reasoning highlighted where one might need to be conscious about ethics (e.g., Buell & Piercy, 2022). Looking at the solutions, independent if treating the items as indivisible or divisible, there are several possibilities. Many of the solutions presented here are similar to what the reasoning children have used in our empirical studies as their first solution attempt (e.g., Eriksson et al., accepted; Hedefalk et al., 2022; Sumpter & Hedefalk, 2023). In most cases, it was only when the teacher explicitly said, "you can cut them", providing a pair of scissors, the items transformed from indivisible items to divisible items. One possible interpretation is that the process of moving between rational numbers and positive integers is not smooth (e.g., Eriksson & Sumpter, 2021). Here, we would like to instead put forward, that working with indivisible items, as presented by Aziz and colleagues (2022), is as important since it presents ethical dilemmas to reason about. Also, it might be easier, when working with indivisible items (i.e., integers), to make ethical reasoning more explicit given that remainders cannot be divided (e.g., Hedefalk & Sumpter, forthcoming). The activity then becomes an opportunity to allow members of the society to explore their mathematical-ethical consciousness.

The main question to address, when you have to allocate items, is "Who should have biscuits?", especially when resources are limited. One way of discussing this from a mathematical modelling point of view it to use the factor of envy-freeness (Aziz et al., 2022). The factor means that one weighs, mathematically, how much one item means for an agent: that the agents perceive the allocation as fair. It is very much a human process. Such weighing can have different strengths, a decision one as developer of mathematical models need to argue for. Here, we use the concepts Information, Coherence, and Engagement to stress different ethical dilemmas in such reasoning (Hedefalk & Sumpter, forthcoming). It can therefore be more 'fair' that one agent gets four biscuits and the other agents get none, if Information is used in a Coherent way, and the agents accept different aspects that could be related to Engagement, for instance seeing it as a envy-free process (e.g., Aziz, et al., 2022). A solution where a dog gets all the biscuits because it is sad it therefore not only a plausible solution, but also both an ethical one as well as a mathematical one (e.g., Hedefalk et al., forthcoming). The challenge then it to help, support, and encourage children and students to express relevant arguments (e.g., Lithner, 2008) that function as a claim (e.g. Toulmin, 2003) for the solutions. This is independent if one defines mathematical reasoning as a collective or individual process (e.g. Eriksson & Sumpter, 2021; Lithner, 2008; Sumpter, 2016; Sumpter & Hedefalk, 2015).

Even in the philosophical solution, the understanding of quantum physics challenge aspects of ethical reasoning. It is, given the design of the quantum computer, difficult to know if you have Information and if it is Coherent. The solution is complex indeed, and one might argue that complex models might be 'more' ethics free. Researchers, in machine learning (ML), among others, has challenged this idea, and in many of the open models, human values are central to the process (Birhane & Sumpter, 2022). The solutions presented here confirm such conclusion: ethics is part of mathematics also at a micro level, not just at a macro-level (e.g., Ernest, 2020; Skovsmose, 2020). The didactical question is how to create spaces for children to work with such ethical reasoning as part of mathematical reasoning (e.g., Hedefalk & Sumpter, forthcoming; Sumpter & Hedefalk, 2023), just as they already do in science education (Griffiths et al., 2017; Öhman & Sund, 2021; Samuelsson 2020; Samuelsson & Lindström, 2020). A didactical implication would then be that, we as researchers and teachers in mathematics and mathematics education, might underestimate the importance of working with '4 shared with 3' as sharing, and not just as division.

References

Aziz, H., Caragiannis, I., Igarashi, A., & Walsh, T. (2022). Fair allocation of indivisible goods and chores. *Autonomous Agents and Multi-Agent Systems, 36*(1), 1-21.

Birhane, A., & Sumpter, D. (2022). The games we play: critical complexity improves machine learning. *Computers and Society.* https://doi.org/10.48550/arXiv.2205.08922

Buell, C., & Piercey, V. (2022). -Ethics in Mathematics: Foreword. *Journal of Humanistic Mathematics, 12*(2), 3-6.

Correa, J., Nunes, T., & Bryant, P. (1998). Young children's understanding of division: The relationship between division terms in a noncomputational task. *Journal of Educational Psychology, 90*(2), 321-329. https://doi.org/10.1037/0022-0663.90.2.321

Eriksson, H., Hedefalk, M., & Sumpter, L. (accepted). The tension between division and fair share. In H. Palmér, C. Björklund, E. Reikerås & J. Elofsson (Eds.), Teaching Mathematics as to be Meaningful – Foregrounding Play and Children's Perspectives: Results from the POEM5 Conference, 2022. Gothenburg.

Eriksson, H. & Sumpter, L. (2021). Algebraic and fractional thinking in collective mathematical reasoning. *Educational Studies in Mathematics, 108,* 473-491. doi.org/10.1007/s10649-021-10044-1

Ernest, P. (2020). Mathematics, ethics and purism: an application of MacIntyre's virtue theory. *Synthese (Dordrecht), 199*(1-2), 3137-3167. https://doi.org/10.1007/s11229-020-02928-1.

Griffiths, M., & Murray, R. (2017). Love and social justice in learning for sustainability. *Ethics and Education, 12*(1), 39-50, https://doi.org/10.1080/17449642.2016.1272177

Hedefalk, M. & Sumpter, L. (forthcoming). Young children's ethical reasoning about sharing: an analytical tool to address different types of arguments. (submitted, under review).

Hedefalk, M., Eriksson, H., Markkanen, P., & Sumpter, L. (2022). 5 year olds in between of sharing and division. *Philosophy of Mathematics Education Journal, 39*.

Lin, D., Wambersie, L., & Wackernagel, M. (2021). Estimating the Date of Earth Overshoot Day 2021. *Nowcasting the World's Footprint & Biocapacity for*.

Lithner, J. (2008). A research framework for creative and imitative reasoning. *Educational Studies in Mathematics, 67*(3), 255-276. https://doi.org/10.1007/s10649-007-9104-2

Öhman, J., & Sund, L. (2021). A Didactic Model of Sustainability Commitment. *Sustainability 13*(6), 3083. https://doi.org/10.3390/su13063083

Samuelsson, L. (2020). Etik i utbildning för hållbar utveckling–Att undervisa den etiska dimensionen av en kontroversiell fråga. [Ethics in education for sustainability – To teach the ethical dimension of a controversial question]. *Acta Didactica Norden, 14*(4), 1-22.

Samuelsson, L., & Lindström, N. (2020) On the Practical Goal of Ethics Education: Ethical Competence as the Ability to Master Methods for Moral Reasoning, *Teaching Philosophy*, 1-22.

Skovsmose (2020). Mathematics and ethics. *Revista Pesquisa Qualitativa, 8* (18), 478-502.

Stemn, B. S. (2017). Rethinking mathematics teaching in Liberia: Realistic mathematics education. *Childhood Education, 93*(5), 388-393. https://doi.org/10.1080/00094056.2017.1367230

Sternberg, R.J. (2012). A model for ethical reasoning. *Review of General Psychology, 16* (4), 319-328.

Suksompong, W. (2021). Constraints in fair division. *ACM SIGecom Exchanges, 19*(2), 46-61.

Sumpter, L. & Hedefalk, M. (2023). När dela lika är olika [When fair share is unequal]. *Nordisk Barnehageforskning, 20*(2), 110–129. https://doi.org/10.23865/nbf.v20.256

Sumpter, L. & Hedefalk, M. (2015). Preschool children's collective mathematical reasoning during free outdoor play. *The Journal of Mathematical Behavior, 39*, 1-10. doi.org/10.1016/j.jmathb.2015.03.006

Sumpter, L. (2016). Two frameworks for mathematical reasoning at preschool level. In T. Meaney, O. Helenius, M.L. Johansson, T. Lange & A. Wernberg (Eds.) *Mathematics Education in the Early Years: Results from the POEM2 Conference, 2014,* (pp. 157-169). Springer.

Toulmin. S. (2003). *The uses of argument*. (updated edition). Cambridge: Cambridge University Press. https//doi.org/10.1017/CBO9780511840005a

Tväråna, M. (2018). Theories of justice among eight-year-olds: Exploring teaching for an emerging ability to critically analyse justice issues in social science, *Nordidactica 2018:4*, ISSN 2000-9879

Vare, P., & Scott, W. (2007). Learning for a change: exploring the relationship between education and sustainable development, Journal of Education for Sustainable Development, 1(2), 191-198.