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# REFLECTING ON MATHEMATICS

## *REFLETINDO SOBRE A MATEMÁTICA*

## *REFLEXIONANDO SOBRE LAS MATEMÁTICAS*

***Edith Schneider***
*(Universität Klagenfurt, Austria)*
*edith.schneider@aau.at*

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ABSTRACT

In our society, which is based on the division of labor, the ability to communicate with experts is a core competence. Reflection plays a central role for non-experts/laypersons in classifying and evaluating expert statements, and in the closely related decision-making process. Reflection is seen as a cognitive ability as well as an attitude. Mathematical concepts and models are applied in many socially relevant areas. For a (better) understanding of these, their fit, effects, limitations as well as their role and meaning, they should be dealt with and reflected on different levels. In the paper, the following levels of reflection are considered in more detail from this social perspective: mathematics-oriented, model-oriented, context-oriented, and subject-oriented reflection. The fact that such reflections can also be integrated into mathematics lessons is demonstrated by means of concrete tasks.

Keywords: reflexive mathematics. mathematics lessons. social role of mathematics.

RESUMO

Na nossa sociedade, que se baseia na divisão do trabalho, a capacidade de comunicar com os peritos é uma competência essencial. A reflexão desempenha um papel central para os não especialistas/leigos na classificação e avaliação das afirmações dos peritos e no processo de tomada de decisões que lhe está intimamente associado. A reflexão é vista como uma capacidade cognitiva e como uma atitude. Os conceitos e modelos matemáticos são aplicados em muitos domínios socialmente relevantes. Para uma (melhor) compreensão dos mesmos, da sua adequação, efeitos, limitações, bem como do seu papel e significado, devem ser tratados e refletidos a diferentes níveis. Neste documento, os seguintes níveis de reflexão são considerados em mais pormenor a partir desta perspectiva social: reflexão orientada para a matemática, orientada para o modelo, orientada para o contexto e orientada para o sujeito. O facto de estas reflexões também poderem ser integradas nas aulas de matemática é demonstrado através de tarefas concretas.

Palavras-chave: matemática reflexiva. aulas de matemática. papel social da matemática.

RESUMEN

En nuestra sociedad, basada en la división del trabajo, la capacidad de comunicarse con expertos es una competencia esencial. La reflexión desempeña un papel fundamental para

las personas no expertas o legas en la clasificación y evaluación de las declaraciones de los expertos y el proceso de toma de decisiones estrechamente relacionado. La reflexión se considera una capacidad cognitiva y una actitud. Los conceptos y modelos matemáticos se aplican en muchos ámbitos socialmente relevantes. Para una (mejor) comprensión de los mismos, de su idoneidad, efectos, limitaciones, así como de su papel y significado, deben tratarse y reflexionarse a distintos niveles. En este documento, se consideran con más detalle los siguientes niveles de reflexión desde esta perspectiva social: reflexión orientada a las matemáticas, reflexión orientada al modelo, reflexión orientada al contexto y reflexión orientada al sujeto. El hecho de que estas reflexiones también puedan integrarse en la enseñanza de las matemáticas se demuestra mediante tareas concretas.

Palabras clave: matemática reflexiva. lecciones de matemáticas. papel social de las matemáticas.

## Introduction

Mathematical activity is not limited to the correct performance of calculations and mathematical operations. It also includes manifold reflections on the meaning of mathematical terms, concepts, representations, procedures, methods, correlations, relationships as well as on the meaning and relevance of mathematics as a whole (cf. Peschek/Prediger/Schneider 2008). Demands from mathematics educators for more “thoughtfulness”, i.e. reflections also in mathematics education are not new. they can be found in numerous mathematics didactic works. The perspectives from which one comes to this conclusion are of different kinds. In the following, the relevance of reflection for (mathematical) education from a societal perspective and relevant orientations for reflection on mathematics will be presented and concretized for the teaching by tasks. The tasks were developed within the research project "Reflecting in Mathematics Education" and have also already been used in the classroom.

## Relevance of Reflecting

An essential characteristic of the functioning our society, which is organized according to the division of labor, is the need to be able to deal with specialized expert knowledge in an appropriate and emancipated manner (see Fischer 2001, 2012). In many questions of public as well as private life and the professional environment, we are confronted with expert statements and must be able to form a judgment about them and to decide or co-decide on the basis of this. Experts here are human beings but also materialized expert knowledge (e.g. books, serious digital media, etc.). Since we can only be experts in a few areas ourselves, we are dependent on those areas in which we are laypersons (non-experts) to obtain expertises from corresponding experts in the field, i.e. to 'communicate' with experts. That is, as laypersons, we must be able to select the appropriate experts for the situation at hand, ask them the appropriate questions, assess their answers, and draw conclusions from them, i.e., make evaluations and, if necessary, make decisions. If necessary, it may also be required to combine different expertises. Fischer (2012, p. 11/12) himself cites the activities of a judge as an example: the judge formulates his judgment on the base of expert opinions of different disciplines, even though he is not an expert in these disciplines. This means that he selects suitable experts (for example psychologists), asks them questions, has to understand, interpret and assess the answers received (expertises), if necessary combine several expertises with each other and finally make a judgment based on them. But we can also get into comparable situations in everyday life: For example, an expert in heating systems presents us with various alternatives that are possible and favorable for us; as laypersons in this field, we have to understand and evaluate his expertise and decide on one (the one that is better suited to our situation). Or: A doctor presents us with possible, alternative treatment methods (e.g. surgery vs. physiotherapy) or different surgical procedures and their pros and cons for the case at hand; we can enquire questions, but as medical laypersons we must then evaluate the doctor's statements and decide on the basis of these for a treatment method/surgical procedure. Analogous situations can be found for professional situations.

Fischer (2001, 2012) sees this *ability to communicate with experts* as a core competence of our society. He makes it clear, however, that this concerns all mature/emancipated people in our society who are capable of acting legally and economically; responsibility in a society based on the division of labor in which we live is highly individualized:

> The legal and economic responsibility of the individual has reached a level never before seen in the history of mankind. (Fischer 2012, p. 13, [translated by the author])

For appropriate and effective communication with experts, and closely related to this the making of decisions based on expertises, Fischer sees, among others, the ability to reflect as extremely relevant: basic knowledge is "*a prerequisite for communicating with experts*" and reflection "*is necessary for the assessing the expertises*" (Fischer 2001, p. 154 [translated by the author]). Reflecting includes, for example, thinking about the meanings of concepts and methods, their possibilities, ranges, and limitations in contexts both within and outside the particular subject. With regard to the development of reflective skills, all subjects should contribute. There will then be subject-specific differences in the focus, due to the characteristics of each subject.

Skovsmose's (1992, 1998) approach to "*critical mathematics education*" is based on the "*formatting power of mathematics*" (1998, p. 197) and its social and political significance for a democratic society. "*Social phenomena are structured and eventually constituted by mathematics*" often as a hidden part. Therefore, strong analytical tools are needed to recognize the role of mathematics, to support the development of a critical capacity towards mathematics and its use and effect in social contexts, and to make citizens "*critical readers of the formatting*" (1998, p. 192). Skovsmose sees reflections as a component of mathemacy and assigns them an important role, a kind of metaknowledge to mathematical knowledge and technological knowledge:

> If mathemacy has a role to play in an education … in trying to develop a democratic competence, then mathemacy must be seen as composed of different competences: mathematical, technological and reflective. And especially: reflective knowledge has to be developed to provide mathemacy with a radicalized power. (Skovsmose 1992, p. 8)

Parallels to this can be found in German-speaking mathematics education in Lengnink's concept of mathematical *'Mündigkeit*', the significance of which lies in a discursive consideration of the relationship between human and mathematics and a critical attitude towards situations containing mathematics. She sees reflecting on and judging mathematics as an important activity in the process towards a mathematical 'Mündigkeit' (Lengnink 2005a, 2005b).

In the OECD's definition of subject-related literacy in the PISA framework, reflective skills are also seen as essential for a "*constructive, engaged and reflective citizen*" of our society; in relation to mathematical literacy:

> Mathematical literacy is an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make well-founded mathematical judgments and decisions needed by constructive, engaged and reflective citizens (OECD 2010, cited in Niss & Jablonka 2014, p 392).

With the requirement to understand the role mathematics plays in the world, making well-founded mathematical judgments and decisions addresses the "*reflective side of mathematics literacy*" (Büscher & Prediger 2019, p. 198) and requires reflecting on meanings, possibilities, limitations of mathematical terms, concepts as well as mathematics itself in extra-mathematical contexts/situations (see also Jablonka 2003).

Reflecting is thus to be seen from different approaches (see above) as essential from a social perspective. It should be an important component of a (mathematical) education in our time. However, the ability to think adequately in life situations about things that are not immediately recognizable and the willingness

to do so does not develop by itself; rather, it must be worked on specifically and occasions must be created that stimulate and support development.

In (Austrian) mathematics lessons, hardly any evidence of reflection intentions can be identified yet. Thus, there seems to be a gap between demand and reality. In the research and development project "Reflecting in Mathematics Education", one focus is to develop interventions in the form of tasks for mathematics teaching that stimulates reflection. These tasks should motivate the students to deal intensively and reflectively with the questions requiring reflections and, if necessary, to produce relevant reflective knowledge on their own. We are concerned with the conception and evaluation of reflection-stimulating tasks that can be integrated directly and continuously into *regular* lessons, into the elaboration and deepening of mathematical terms and concepts being part of the curriculum. For each task, the intention associated with the development of the task in terms of reflection, necessary prior mathematical knowledge, and suggestions for the form/method of instruction are given. This is intended to promote/facilitate immediate use in regular lessons by the teacher. In addition to this developmental focus, the project focuses on questions concerning the gathering of practical teaching experience with the use of these tasks, both in terms of construction and teacher behavior, as well as the students' handling of these tasks, the reflection processes that take place, and the reflective knowledge that is produced in the process. This paper will focus on that part of the project that concentrates on possible tasks for reflections on mathematics.

**Reflection means …**

According to Duden, one of the conceptual meanings of reflection is: "2. Thinking about; deliberation, examining consideration" [translated by the author] (https://www.duden.de/rechtschreibung/Reflexion, 31/01/2023); in the online Cambridge Dictionary, 'reflection' includes: "serious and careful thought" (https://dictionary.cambridge.org/dictionary/english/reflection, 31/01/2023). In our project, "reflection" was clarified by the following definition in order to be able to specify it for mathematics in the following:

> Reflection ... means thinking about characteristics, connections, relationships, effects or meanings that cannot directly be read from the given fact. (cf. Schneider 2020)

We thus assume a broad concept of reflection and understand reflection as thinking about things that are not in the foreground of object language. This "definition" follows Fischer's characterization of reflection (cf. Peschek 2005), but becomes more concrete with respect to the objects of thought.

Reflecting is thus to be understood as a thinking activity that is initially formulated independently of the subject. From a social perspective, it is on the one hand necessary to develop this thinking activity in order to be able to think adequately about concepts, connections, effects, meanings, etc. in situations. On the other hand, reflecting should also be understood as an attitude. It should be considered important and necessary for the individual (as well as for the collective) to form their own picture of situations, statements, etc. by reflecting on them, to want to asses and evaluate them, to want to understand something more precisely, ... In the sense of Heymann:

> "It is not enough to have cognitive skills and related knowledge; one must also develop the willingness to make use of them." (Heymann 1996, p.67; see also Peschek, 2005, p. 60 [translated by the author])

Both views of reflection, reflecting as an activity as well as an attitude, are of (social) relevance. This means that reflections should be made more explicit in the classroom, they should be given appropriate space and, above all, importance. On the one hand, reflection should be practiced by means of appropriate tasks, and on the other hand, a corresponding attitude should be developed through the continuous working with reflection tasks. On the one hand, reflection should be practiced by means of suitable tasks and, on the other hand, a corresponding attitude should be developed through the continuous processing of reflection tasks. Thereby, reflecting *on mathematics* makes a double

contribution. It contributes to the development of a general ability to reflect and attitude, which can be relevant in all subjects and life situations. And second, mathematics-specific reflection can also foster a deeper understanding of mathematics, mathematical content and concepts, their effect, function, and relevance.

What should be reflected on in and with mathematics requires a differentiation of the concept of reflection focusing on mathematics. In the following chapter, this will be illuminated from a social perspective and illustrated by concrete tasks for math lessons.

**Reflection focusing on mathematics**

Depending on the respective approaches and perspectives from which the relevance of reflections is considered, different categories of reflection can be found in the literature. For our project, besides considerations by Fischer (2001), Peschek (2005) and Lengnink (2005a), especially the levels of reflection by Skovsmose (1992, p. 8/9), which he later summarized in four categories (1998, p.199/2000), were guiding. We distinguish four types of reflection depending on the focus of the reflection (see also Schneider 2020). The first three types of reflection coincide in designation and intention with those of Skovsmose's categories, the fourth emphasizes more strongly the relationship between humans and mathematics than seems to be the case with Skovsmose. A specific concretization of the reflection types was necessary with regard to our focus on task development and the embedding of these tasks into regular teaching by the teachers themselves:

> *Mathematics-oriented reflection*: thinking about mathematical properties of mathematical concepts (mathematical objects, representations, procedures, theorems, etc.) and about mathematical relations within or between such concepts.
>
> *Model-oriented reflection*: Thinking about relations between mathematical concepts and inner-mathematical, but above all extra-mathematical situations.
>
> *Context-oriented reflection*: Thinking about the effects of mathematical concepts in our world.
>
> *Subject-oriented reflection*: Thinking about the importance and relevance of knowing mathematical concepts and topics for oneself or for communities or for society.

It is obvious that mathematical concepts and models are used and play an important role in numerous socially relevant fields of our (life)world. For the layperson explicitly recognizable or also more or less hidden. In order to recognize and (better) understand the social relevance of these inherently abstract mathematical concepts and models, as well as to be able to assess and evaluate them (also with regard to decision-making), a consideration and evaluation of them should take place on different levels. The models and concepts should be reflected in their direct applications (as models of a extra-mathematical situation), in their societal function (societal purpose of a mathematization) and especially in their relevance and importance for the individual himself as well as for the collective (benefit for oneself, benefit for society). The first one is part of model-oriented reflection, the second one belongs to context-oriented reflection, and the third one to subject-oriented reflection. Mathematics-oriented reflection plays a rather subordinate role from this perspective. In the following, these different levels of reflection will be discussed in more detail. The fact that reflections on the social relevance of mathematical concepts and models can also be implemented in mathematics lessons is demonstrated exemplarily by concrete tasks. The tasks are taken from a pool of tasks which was developed for different mathematical contents of secondary level (in Austria grade 5 to 12) and types of reflection in the context of the project. The pool of tasks currently comprises more than 80 tasks. Each of these reflection tasks also contains hints for an adequate instructional use (intention of the reflection task, required prior mathematical knowledge, suggestion for instructional form/method). The development of the tasks has gone through several quality assurance loops (e.g. testing of selected tasks in class).

## Mathematics-oriented reflection

The focus of mathematics-oriented reflection is on thinking about mathematical properties, mathematical concepts and mathematical relationships within and between such concepts. It will play a rather subordinate role with respect to an emancipated approach to (mathematics-containing) expertise and mathematics-containing situations in the sense described above. Of course, reflection on the validity of specific mathematical rules or on the mathematical correctness of applied algorithms will be important for the development of a reflective basic knowledge of mathematics (Fischer 2012) or technical and mathematical knowledge (Skovsmose 1992). However, the evaluation of expert statements that laypersons have to make does not primarily refer to the mathematical correctness of the statements. Mathematics-oriented reflection can be more significant from a social perspective if it places its focus on thinking about what is characteristic of a particular mathematical topic. Reflecting on the global/central ideas of a mathematical topic, on meta-concepts (for a bundle of strategies, actions, procedures, techniques, questions, ways of thinking and working) characteristic of a mathematical topic. Insights of this kind could be helpful for the classification of questions or problems within mathematics as well as their relevance (referring to ability to communicate: "Which mathematical topic/area is relevant for my problem?"; "What kind of answers can I (not) expect in this mathematical area?").

## Model-oriented reflection

The focus of model-oriented reflection is on the relationship between mathematics and the world. Reflecting on mathematical models of extra-mathematical situations and their fit (being an adequate description of the present situation), limits, effects, and implicit assumptions for the concrete situation can contribute to recognizing and adequately understanding the role of mathematics for our world. Questions such as: In what way does the mathematical model (chosen by the expert) describe or create realities? What limitations does it impose? In what way are parameters taken into account, others neglected? What contextual consequences/effects does the modeling have?, are relevant here. Reflection on this can be seen as a prerequisite for an emancipated, discursive engagement with mathematics-containing situations of our (life)world in the sense of Lengnink or a critical reading of the formatting power of mathematics in the sense of Skovsmose (see above) as well as for the assessment and evaluation of mathematics-containing situations in extra-mathematical contexts (among others for decision making as a non-expert).

In the following, two concrete tasks for mathematics lessons that could trigger model-oriented reflections:

*TASK 1: Average salaries* (cf. Schneider et al, 2021)

The following table shows the arithmetic mean and the median of the monthly gross salaries of employees (total and separate for women and men) in 2017:

| ***Monthly gross salaries of the employees in 2017 (in €)*** | | | |
|---|---|---|---|
| | *total* | *women* | *men* |
| *arithmetic mean* | 2.780,- | 1.950,- | 3.770,- |
| *median* | 2.250,- | 1.690,- | 3.250,- |

**Table 10**: *TASK 1* about *Average salaries*. Personal file

Why does the average of monthly gross salaries of employees give significantly different values when modelled by arithmetic mean and by median?

*TASK 2: Different models* (cf. Schneider et al, 2021)

Natural growth or decay processes (e.g. spread of epidemics, growth of water lilies, growth of bacteria or radioactive decay, death of bacteria or bark beetles, etc.) are often modelled by exponential functions.

These models usually describe the situations only approximately, but sufficiently accurately for the respective purpose.

The situation is different, for example, when modelling the return on capital by means of compound interest: the exponential model must fit exactly here, even small deviations are not conceivable here and would also be unacceptable.

How can this difference be explained?

Does it only occur with exponential functions or also with other types of functions?

The intention of task 1 "Average salaries" is to reflect on the effect of two models of average (arithmetic mean and median), on a list of data (gross salaries of employees). The two models provide quite different (model-)results, specifically: quite different average salaries. To recognize that there is an asymmetric distribution of gross salaries of employees and in which way this asymmetry affects the position of arithmetic mean and median is necessary for an appropriate interpretation, assessment and evaluation of the given results. The reflective knowledge (product) generated by reflecting (process) is furthermore useful with regard of deeper understanding as well as with regard of sustainability.

The reflection in task 2 "Different models" should aim at the question in which way reality is modeled by a mathematical concept. The intention is to recognize the different type of the two models. On the one hand, a phenomenon in our environment is approximately described (descriptive model), on the other hand, a rule/norm to be followed is created/constructed by humans (normative model). This difference is not only observable in exponential models, it is in any case essential for the evaluation of a mathematical model. Therefore, not only the reflection process is of interest here, but in any case also the reflection knowledge developed with it. Understanding the role of mathematics in our world on the one hand as a means to describe and recognize phenomena and on the other hand as a means to construct norms (cf. Heymann 1996, p.184) is in the center. The two tasks clearly address components that Skovsmose sees as reflective knowledge in relation to modeling: "*reflective knowledge, to be interpreted as a more general conceptual framework, or metaknowledge, for discussing the nature of models and the criteria used in their constructions, applications and evaluations*" (Skovsmose 1990, p.765).

**Context-oriented reflection**

Context-oriented reflection likewise illuminates the relationship between mathematics and the world. While model-oriented reflection aims at the fit of mathematical models, context-oriented reflection, however, is about thinking about the effect of mathematics in our world (see also Skovsmose 1998, p. 199) It focuses on mathematizations in our world and on thinking about their (often hidden) social function. Questions of the following kind can trigger context-oriented reflection processes (exemplary mentions): What is the purpose of using a specific mathematical concept in the given context (or in other social contexts as well)? What effect does a particular mathematization have, what advantages and what disadvantages does it bring? How does the use of a certain mathematical concept influence our ideas of social situations, of our (life-)world?

*TASK 3: Why a measure of average?* (cf. Schneider et al 2021)

The students of a class receive a monthly allowance of 50 Euro on average from their parents.

Which advantage does the specification of a measure of advantage have compared to the specification of all data, which disadvantage does it have?

*TASK 4: Formulas - why?* (cf. Schneider et al 2021)

Formulas are of great importance both within mathematics and in its applications. In mathematics lessons we have learned about formulas in many different contexts.

For what purpose are formulas used? Explain with examples.

The tasks 3 and 4 are examples of how context-oriented reflection tasks could look like, which can be directly integrated into mathematics lessons. Even in a conventional mathematics lesson. The intention of task 3 is to use a concrete situation to consider what effect measures of average have in our world, what would be lost in the present context if the mathematical concept of average were not available. It would be interesting to consider the context differentiated for mean, median and mode. Reflecting on a concrete situation can serve as a starting point and be extended to other contexts in order to recognize the advantages of a specific mathematical concept for our society, for our (living)world. Task 4 is intended to encourage thinking about the (social) function of common mathematical objects (formulas), about the purpose for which formulas can be used (in our world).

Initiating reflection processes as illustrated in tasks 3 and 4 in the mathematics classroom can and should serve as a basis for context-oriented reflection outside the classroom as well. Context-oriented reflection takes into account a number of aspects that Fischer cites in his description of the field of reflection: *"....reflection (What is the meaning of the terms and methods, what do they accomplish, what are their limitations)."* (Fischer 2001, p. 154 [translated by the author]). Also, goals of a conception of mathematical literacy that has its focus on the evaluation of mathematics are described by Jablonka (2003, p.89) as *"... To educate for an awareness of applications that affect society, and to develop a consciousness of limits of reliability of mathematical models."*. While the last part points more towards model-oriented reflection, the first part clearly addresses context-oriented reflection. While Lengnink (2005a) combines model- and context-oriented reflection in her approach, we follow Skovsmose (1998). By separating them, the different focus of the two types of reflection can be emphasized more clearly.

**Subject-oriented reflection**

Subject-oriented reflection is about thinking about the importance and relevance that knowledge of mathematical concepts and content has, on the one hand, for oneself and, on the other hand, for a community and for society. Mathematical content can mean a very concrete content (e.g. arithmetic mean), but also a whole content area (e.g. descriptive statistics). Subject-oriented reflection thus brings into play the relationship between the human(s) and mathematics. Both the individual relationship to mathematics and the collective relationship to mathematics are of interest. At the core of such reflections are questions of the type:

- What does a mathematical content mean to me personally, what does it mean to me as part of a community (e.g., family, class, etc.) or as a member of our society? What are the benefits for me of being familiar with a mathematical content? Where can I use appropriate mathematical knowledge and skills now or in my future life?

- What are the benefits of being familiar with a mathematical content for communities (e.g. family, friends, etc.), for our society? What problems/difficulties/disabilities might occur if this familiarity were not present.

Reflections on the meaning and importance of mathematical content can arise unplanned from a learning situation ("Why do we have to learn this?"), but they should also be initiated by appropriate tasks (see also Peschek 2005).

*TASK 5: Plea for/against descriptive statistics* (cf. Schneider et al 2021)

Imagine that the Ministry of Education is planning to remove descriptive statistics from curricula.

Write a letter to the editor of a daily newspaper in which you explain and justify your opinion of the Ministry of Education's plans.

Task 5 is intended to be an "evaluative" consideration of the content of descriptive statistics. It is about thinking about the importance and the benefit of the familiarity with descriptive statistics for oneself, for one's mathematical education as well as for our society as a whole. It does not necessarily follow from the importance or even the insignificance for oneself that one estimates it to be so for society as well. It is possible to evaluate the relevance for the individual and the collective differently. In this case it should not be fixed from the task whether pro or contra points of view or a balanced point of view to the descriptive statistics are taken. On the one hand, one's own assessments and their justifications seem to be important, as well as a confrontation/discussion of one's own considerations with those of others.

For Fischer, education takes place in the process of dealing with the *"question 'What do the contents mean for me, what do they mean for society, what do they mean for us as a community, ...'"* (Fischer 2001, p. 158 [translated by the author]). Education is seen as a social process of engaging with and negotiating meaning and significance (of mathematical content). Becoming aware of the relevance and importance of a mathematical content for oneself and especially for society is a central element in communicating with experts or making decisions under uncertainty.

## Conclusion

In the project, the intention of the task development is to create tasks that can be continuously integrated by teachers into the regular mathematics lessons and to focus on the curricular mathematical content and not on generic topics (like fairness, etc.). Comments for teachers (like description of the intention of the task) should support the integration in class. The aim is to practice reflection through continuous engagement with reflection tasks and thus to develop general reflection skills and reflective attitude, which are important for the individual as well as the collective, in order to be able to use them flexibly in variable (life) situations.

Within the project, reflection tasks with different focus (see tasks 1-5) were used in different secondary classes. First, tasks were tested in terms of comprehensibility and feasibility (from a student and also a teacher perspective) in class; seven teachers participated in this pilot testing, each with one class. In a second phase, the focus was on the question of how teachers deal with such tasks and the comments provided on the tasks on the one hand, and how students deal with such tasks on the other hand. Six teachers from three different schools, each with one class, took part in the study; they used tasks selected by them from the task pool. In the third phase, the focus of the study was on the reflection processes of the students (4 classes from two different schools) during the working with reflection tasks. None of the participating classes had much experience with such tasks so far; the latter was especially true for tasks with a context-oriented and personality-oriented focus. Accompanying the use of tasks, we were able to make a lot of observations and gain experience, of which I would like to pick out a few that seem interesting to me in the context of this paper. These are not results of a systematically conducted empirical study, but rather initial findings that could be gleaned from observations of teaching and reports and protocols.

The students were to a large extent very willing and motivated to engage in reflection tasks. They also explicitly expressed their interest in such tasks ("I found it very good, we had to really think about it ").

The teachers dealt with the reflection tasks in a somewhat more distanced way. They often saw reflection tasks as something additional to regular lessons and less as tasks that could replace other content/tasks and be integrated directly into lessons. But overall, they rated reflection tasks positively, with student motivation also being an important motivator for teachers.

The teachers were satisfied with the students' reflection processes more quickly than we, the project team. This means that the results were assessed as (very) satisfactory by the teachers more quickly than

by us. They also did not usually see a need for follow-up on content, even when deficiencies in student performing were seen by the teachers. The teachers could have evaluated the results of the reflection tasks in a more differentiated way and used them more for further work and further discussions.

The positive attitude of the students towards the reflection tasks is very promising for a continuous integration of tasks aiming at a reflection of mathematical concepts and models with respect to their applications in extra-mathematical situations (model-oriented reflection), social function (context-oriented reflection) and their meaning and importance for oneself as well as a member of society and for society as a whole (subject-oriented reflection). Interest and willingness to deal with such questions seems to be given.

Observations on how teachers deal with such reflection tasks provide indications that the training of mathematics teachers is challenged here. On the one hand, with regard to the importance that should be given to such reflection tasks (i. e. in the sense of Fischer: "*a reduction of the demands with regard to operating and an increase of the demands with regard to reflection*" (Fischer 2001, p. 155 [translated by the author])) on the other hand also with regard to the use of reflection processes and results/products of reflections for an adequate understanding of the role mathematics plays in and for our (life)world and its usefulness and significance for the individual as well as the collective.

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