CONFLICTING MATHEMATICAL WORLDVIEWS
EVEN MORE CHALLENGES FOR MATHEMATICS PRESERVICE TEACHERS

VISÕES DE MUNDO DA MATEMÁTICA CONFLITANTES
Ainda mais desafios para os professores de matemática de formação inicial

CONFLICTOS EN LA VISIÓN MATEMÁTICA DEL MUNDO
Aún más retos para los futuros profesores de matemáticas

Robin Göller
(Leuphana University Lüneburg, Germany)
robin.goeller@leuphana.de
Recibido: 09/07/2023
Aprobado: 09/07/2023

ABSTRACT

University mathematics education and civic education through mathematics emphasise
different aspects of the mathematical worldview. The present study qualitatively analyses 21
mathematics students’ mathematics-specific intrinsic motivation for choosing a mathematics
study programme referring to different aspects of their mathematical worldview before the
beginning of their study. The results show that many preservice teachers enjoy schematic
aspects of mathematics, which does neither correspond to the type of mathematics taught at
universities nor to civic education through mathematics. These findings contribute to
explaining demotivation and frustration of preservice teachers with university mathematics
as well as challenges for implementing civic education through mathematics in classrooms.

Keywords: mathematical worldviews. mathematics-specific intrinsic motivation. university
mathematics education. civic education through mathematics.

RESUMO

A educação matemática universitária e a educação cívica por meio da matemática enfatizam
diferentes aspectos da visão de mundo da matemática. O presente estudo analisa, qualitativamente, a motivação intrínseca específica da matemática de 21 estudantes de
matemática na escolha de um programa de estudos de matemática, referindo-se a diferentes
aspectos de sua visão de mundo da matemática antes do início dos estudos. Os resultados
mostram que muitos professores em formação gostam de aspectos esquemáticos da
matemática, o que não corresponde ao tipo de matemática ensinada nas universidades, e nem
à educação cívica por meio da matemática. Esses resultados contribuem para explicar a falta
de motivação e a frustração dos professores em formação com a matemática universitária,
 bem como os desafios para a implementação da educação cívica por meio da matemática nas
salas de aula.
Palavras-chave: visões de mundo da matemática. motivação intrínseca específica da matemática. educação matemática universitária. educação cívica por meio da matemática.

RESUMEN

La educación matemática universitaria y la educación cívica a través de las matemáticas hacen hincapié en diferentes aspectos de la visión del mundo matemático. El presente estudio analiza cualitativamente la motivación intrínseca relacionada específicamente a las matemáticas de 21 estudiantes de matemáticas para elegir un programa de estudios de matemáticas haciendo referencia a diferentes aspectos de su visión del mundo matemático antes de comenzar sus estudios. Los resultados muestran que a muchos futuros profesores les gustan los aspectos esquemáticos de las matemáticas, que no se corresponden ni con el tipo de matemáticas que se enseña en las universidades ni con la educación cívica a través de las matemáticas. Estos resultados contribuyen a la explicación de la falta de motivación y frustración de los futuros profesores con las matemáticas universitarias, así como los retos que plantea la aplicación de la educación cívica a través de las matemáticas en las aulas.

Palabras clave: visiones matemáticas del mundo. motivación intrínseca específica de las matemáticas. educación matemática universitaria. educación cívica a través de las matemáticas.

Two challenges: The transition from school to university and mathematics education as civic education

The transition from school to university in mathematics is associated with several challenges (e.g., Gueudet & Thomas, 2020) such as more abstract and formal modes of mathematical thinking at university, including a stronger focus on deductive proof, as well as different institutional cultures regarding the teaching and learning of mathematics (e.g., Gueudet & Thomas, 2020). For upper secondary preservice teachers with mathematics as a subject, who in Germany typically attend courses on abstract university mathematics together with students majoring in pure or applied mathematics (mathematics majors), these challenges often result in frustration (Göller & Gildehaus, 2021) or questions about the relevance of university mathematics for their later profession (Gildehaus & Liebendörfer, 2021a). Preservice teachers report a higher interest in school mathematics and a lower interest in university mathematics as well as a lower interest in proof and formal representations than mathematics majors (Ufer et al., 2017). In turn, such an interest profile predicts study demotivation and dissatisfaction (Kosiol et al., 2019).

Besides these mathematical challenges at the transition from school to university, mathematics education as whole is facing challenges due to an increasingly complex world that gives mathematical models an increasingly important role in social and political decision-making. Recent examples for this are climate change, pandemics, or sustainability. This means that mathematics education overlaps with civic education, aiming at educating responsible citizens who are able to actively participate in society and critically reflect on social and political decisions. Therefore mathematics education should at least (or maybe even needs to) comprise (normative) modelling (Gildehaus & Liebendörfer, 2021b; Pohlkamp & Heitzer, 2021; Steffensen et al., 2021) and be critical (Skovsmose, 2020; Steflitsch, 2021). This means that preservice mathematics teachers (as all mathematics teachers) face the challenge to implement such a mathematics education in their (future) classrooms.

Elaborating on these challenges the present paper aims at identifying students’ mathematics-specific (intrinsic) motivation for choosing mathematics as a study subject in terms of different mathematical worldviews (Grigutsch & Törner, 2002). Building on this, it will be discussed how these mathematical worldviews are related to those of university mathematics and those of civic education through mathematics.
Theory

Study and career choice motivations of future teachers

In light of the importance of the teaching profession for the future generation and society as a whole, study and career choice motivations of future teachers have been studied frequently in recent years (Fray & Gore, 2018; Göller & Besser, 2021; König et al., 2018; McLean et al., 2019; Wach et al., 2016; Watt et al., 2012). In current surveys, such study and career choice motivations of future teachers are often categorised on the basis of an expectancy-value theory of motivation (Eccles & Wigfield, 2002; Watt et al., 2012). Expectancy-value theory theorises that students’ motivation for an upcoming task or here their study programme can be explained by their expectancy how well they will do on it (e.g., their perceived ability for the study programme), their perceived enjoyment (intrinsic value, such as subject or pedagogical interest), personal importance (attainment value), and usefulness for other goals (utility value, such as a secure income), as well as by perceived negative aspects (cost) associated with the study programme (Eccles & Wigfield, 2002).

Study and career choice motivations of future teachers are an important predictor for individual success in university studies and profession: Especially intrinsic study and career choice motivations are associated with students’ strategy use and study satisfaction (Wach et al., 2016), with lower burnout and higher career optimism (McLean et al., 2019), as well as with learning goal orientation and general pedagogical knowledge (König et al., 2018). Furthermore, in the long term they are also positively related to pedagogical competence and satisfaction in the teaching profession (Hanfstingl & Mayr, 2007). In absolute terms, intrinsic study and career choice motivations are the most pronounced among preservice teachers (Fray & Gore, 2018; König et al., 2018; Wach et al., 2016; Watt et al., 2012). However, this is also true for students of study programmes beyond teacher training, for which the role intrinsic study choice motivations is even more prominent (Göller & Besser, 2021).

Mathematical worldviews

Given the significance of intrinsic study choice motivations, the present paper aims at identifying mathematics-specific aspects students’ intrinsic motivation refers to. In other words, this paper aims at identifying what students like (so much) about mathematics that they choose mathematics as a study subject. Grigutsch & Törner (2002) identified four aspects as being essential for people’s view and understanding of the nature of mathematics and shape their mathematical worldview:

- The process aspect highlights the constructive character of mathematics, being an active process of discovery, experimentation, and thinking about problems.
- The application aspect emphasises the usefulness and practical applicability of mathematics for society and everyday life.
- The formalism aspect identifies logical and objective thinking, manifested in rigour, exactness, and precision as the essence of mathematics.
- The schema aspect understands mathematics as a “toolbox” and set of formulas for solving tasks.

While process and application aspect are summarised as dynamic view of mathematics, regarding mathematical knowledge as being constructed through problem-related discovery and real-world applications, formalism and schema aspect are seen as static view of mathematics, regarding mathematics rather as a collection of existing knowledge and procedures (Grigutsch & Törner, 2002).

University mathematics, as presented above, rather seems to emphasise the formalism aspect as well as (perhaps to a somewhat lesser extent) the process aspect (Gueudet & Thomas, 2020). With regard to civic education through mathematics a rather dynamic mathematical worldview is addressed which emphasises the process and application aspect (Gildehaus & Liebendörfer, 2021b; Pohlkamp & Heitzer, 2021; Skovsmose, 2020; Steffensen et al., 2021; Steflitsch, 2021).
Research questions

The present study aims at identifying students’ mathematics-specific intrinsic motivation for choosing mathematics as a study subject. Thereby, the four presented aspects of the mathematical worldview (Grigutsch & Törner, 2002) shall serve as framework for the following research question:

• To which aspects of mathematics does the intrinsic motivation of students refer to choose mathematics as a study subject?

Methods

To answer this research question, interview data of a total of 21 students were analysed. The sample comprised 14 upper secondary mathematics preservice teachers (T, 11 female, 3 male) and 7 students (3 female, 4 male) who had chosen a study programme in pure mathematics (mathematics majors, M). The interviews were conducted about three weeks before the beginning of their university studies. For this paper answers referring to the following two questions were analysed: (1) How come you decided to study for a mathematics degree programme? (2) Why mathematics, what do you like about it? The data were analysed with a concept-driven (deductive) and data-driven (inductive) coding approach (Kuckartz, 2019). The four presented aspects of the mathematical worldview were used as concept-driven categories. These deductive categories were further elaborated inductively.

Results

The data analysis revealed that the mathematical worldview to which students’ intrinsic motivation to choose mathematics as a study subject referred, could be roughly mapped by regarding process and schema aspect as well as application and formalism aspect as opposite poles of a respective dimension. Figure 1 gives an impression of this ordering of the aspects as well as the interviewees’ positioning within this framework. This positioning visualises students’ mathematical worldview coded in the data, depending on whether their mathematics-specific intrinsic motivation rather referred to calculating or to puzzling (schema – process aspect, vertical axis), or rather to real-world applications of mathematics or to precise, logic inner-mathematics relationships with unambiguous right and wrong (application – formalism aspect, horizontal axis). In the following, these aspects as well as their respective polarity will be elaborated on in more detail. Furthermore, the data-driven coding revealed that sense of achievement and perceived competence were closely linked to mathematics-specific intrinsic motivation. These interrelationships will be explained in more detail hereafter.
In between process and schema aspect

The process aspect was coded for passages where students stated that they like mathematics when they puzzle over problems or because they can discover new things. The following excerpt is a prime example of this aspect:

T: Just what I said earlier that somehow you always have to puzzle, you sit and try to solve it. And maybe it doesn’t work right away, but at some point, you have this “aha” moment. And I think that’s somehow exciting. [...] I think mathematics is simply fascinating. What kind of things there are, what new things you can discover.

Schema aspect was coded for passages where students stated they like performing calculations and mathematical procedures. Indicators of this aspect were on the one hand when students stated that they like calculating or tasks where they knew exactly which formula they had to apply, and on the other hand when students stated that they liked the routines of these procedures which they could perform (quickly) without any difficulties. Examples of this aspect are given in Figure 1 and by the following excerpt:

T: We also had programmable graphic calculators. It was pretty quick. So as far as graphs are concerned, how they run, inflection points, zeros and so on and so forth, as I said, there were fixed formulas, and you calculate it with them. And that was the most important thing for me. That I know how to calculate it.

The idea that process and schema aspect can be considered as opposite poles of a dimension of mathematics-specific intrinsic motivation is given in the data. This dimension describes the extent to which students prefer routine tasks with a predefined procedure for providing a (or the predetermined) solution, or rather problem-solving tasks that require puzzling and allow for new approaches and insights. The following excerpt gives an insight into how positioning within this dimension is negotiated, in this case towards the process aspect:
T: There were more tricky tasks. Not those where you just have to calculate something, but rather those tasks where you really have to puzzle. I liked those better. Because the rest was mostly just penwork anyway, your hand hurt afterwards, where you don’t have to think a bit.

In this case the process aspect, indicated here by “tricky tasks […] where you really have to puzzle” was preferred to the schema aspect, represented here by tasks that only require calculating and “penwork” without thinking.

Other students rather liked calculating and “penwork”, and thus the schema aspect (see also Figure 1 for examples). The following excerpt gives another insight into how positioning between process and schema aspect is negotiated, in this case rather towards the schema aspect:

T: I liked everything where I could calculate. I liked that more than things where you only thought something through theoretically. […] I like to calculate. Sometimes you have a task where you have to calculate somehow, but you don’t know WHAT you have to calculate at first. So, I like to calculate, I also like to puzzle. I just don’t like to do things where I can NOT find the approach.

In this case the student states that she likes both calculating and puzzling, by which she positions herself somewhere between process and schema aspect. However, the emphasis on her preference for calculating and that she likes puzzling only when she finds an approach, positions her rather towards the schema aspect. In Figure 1 this is represented by the topmost point of the five points above the schema aspect.

Figure 1 also shows that mathematics-specific intrinsic motivation (for choosing mathematics as study subject) for 7 of the 14 preservice teachers mainly referred to the schema aspect of mathematics. Process aspect was dominant for only 3 of the 14 preservice teachers. Of the mathematics majors, intrinsic motivation referred to the schema aspect for only one student, while all other math majors tended towards the process aspect or indicated no preference here.

**In between application and formalism aspect**

Formalism aspect was coded when interviewees stated that they like mathematics because of its logic or its clear structure. Formalism aspect occurred in two forms. The first form identifies mathematics as “universal language” which allows a “beautifully organised clear structure” emphasising the closedness and consistency of mathematics as given by the two mathematics majors in Figure 1 respectively the following excerpt:

M: The fact that ultimately it is pretty much the most universal language imaginable. What is the best way to summarise this? Well, simply this wonderfully beautifully organised and clear structure. Because ultimately, throughout history, how many errors have appeared in mathematics? Practically none at all. Because immediately, when an error appears, you can correct it, and you can correct it from WITHIN, without needing any external experiments.

The second form of the formalism aspect emphasises the logic, unambiguousness, and precision of mathematics, in the sense that it is “logic” for oneself without discussion about right or wrong, especially in comparison to other study subjects (cf. Figure 1):

T: I don’t know exactly what I enjoyed about it [mathematics], but I think it’s simply this unambiguousness. That you know I have a task and there is one solution. And this solution is either correct or I did it wrong. Then I have to do it all over again. But I think that’s actually better than having to write an interpretation in German and then being told at the end: “No, the artist meant it completely differently”, and then I wonder how my German teacher would know that. And in maths it’s just clear. And there, I think, it’s more a case of saying, “I’m enthusiastic about this. That I know I’ve done it right or wrong,” and not something in between.
This second form of the formalism aspect is connected to the schema aspect in the sense that predetermined procedures provide predefined solutions.

T: I need to know what is required from me, how I have to write it down. Then I can do it. And this was the beauty. Because in maths you know how an equation needs to be written down, you know how it is modified, with which rules you want to do it. And if it says “plus” that means plus. For me, maths is precise.

Application aspect was coded when interviewees stated they like mathematics for its real-world applications. Examples of coded sequences for the application aspect are given in Figure 1 and by the following excerpt:

T: In any case, that pupils understand that maths also has a purpose. That they don’t always think: “I’ll never need this again in my life anyway.” I mean, well, maybe some things are like that. But you can still calculate a lot of applications with maths. So, we often did something like calculating the statics of a bridge. And I actually find that quite interesting. Stochastics is also very application oriented. And that pupils realise: “Okay, I don’t just have to be able to calculate plus and minus, and that’s all I really need”, that’s not really it. So that pupils enjoy maths. That they like going to class and that they understand why they should learn it in the first place. That they see a sense in it.

Application aspect, as can be seen in this quote, occurred in two forms in the data. On the one hand it occurred as intrinsic motivation for the interviewed students themselves, indicated here by “I actually find that quite interesting”. In this case the application aspect gives meaning and purpose to mathematics which makes it interesting for the interviewed students. This aspect referred predominantly on mathematics-specific intrinsic motivation for only one and partly a second of the interviewed students (for some others it played a role, albeit not a pronounced one).

On the other hand, application aspect occurred as possible motivation for others, such as e.g., future pupils of the interviewed preservice teachers. In the quote above, this is indicated for example by “that pupils understand that maths also has a purpose”, “that pupils enjoy maths” or “that they see a sense in it [mathematics]”. Such reflections on possible motivational effects of the application aspect were also found among students who themselves have a rather formalistic mathematical worldview:

T: Maybe it’s easier to hook pupils by saying: “Yes, that’s how it is. Now think about it: How does a navigation device or something work? That it all has something to do with mathematics. In this way, you can perhaps get pupils more interested in mathematics than if it is simply done formally. […] But I also think that you shouldn’t forget that mathematics is a formal matter. And that I have to learn certain theorems and definitions, or the rules at school, through frontal teaching.

In this case, the application aspect (“How does a navigation device […] work?”) was rather seen as a motivational tool to “hook pupils” or “get pupils more interested in mathematics” while for the interviewed student herself, mathematics was rather “a formal matter”. Thus, this excerpt indicates how the idea that application and formalism aspect can be considered as opposite poles of a dimension of mathematics-specific intrinsic motivation is represented in the data. The following excerpt gives an insight into how positioning within this dimension was negotiated:

M: Many people have said to me: “What you want to study maths for a bachelor’s degree? That’s much too dry! There’s no connection to reality.” I didn’t need that at school, and I don’t need it today either. […] Mathematics is enough for me. It’s enough for me when I’ve accomplished a proof on my own, even if it’s only to prove that you can apply the factor rule, [that makes me] happy as anything! That means I don’t really need the real-world reference at all.

While intrinsic motivation (“happy as anything”) strongly referred to the formalism aspect (accomplish a prove) here, application aspect (“real-world reference”) was explicitly described as subjectively unimportant.
Overall, the application aspect played a rather subordinate role for the mathematics-specific intrinsic motivation of most of the interviewed students (cf. Figure 1). Especially among the mathematics majors in this sample, intrinsic motivation referred rather to the formalism aspect and almost not at all to the application aspect. Among the preservice teachers, both application and formalism aspect were represented (cf. Figure 1).

**Sense of achievement and perceived competence**

The data-driven coding identified sense of achievement and perceived competence as important categories associated with mathematics-specific intrinsic motivation. For some students, it seems that this perceived mathematical competence was the decisive factor for their study choice (e.g., “I wanted to study [a subject] I was always good at in school”). This means that with a sufficiently high perceived competence, sense of achievement or experiences of success in mathematics are to some extent expected and perceived as mathematics-specific intrinsic motivation in itself for some students.

T: I have always enjoyed mathematics. Especially this principle that you have so much sense of achievement, I would say. That’s quite appealing. And I think the school grade also plays a role. That was the deciding factor [for my study choice], I’d say.

Sense of achievement is also closely associated with the aspects of mathematical worldview. For process aspect e.g., sense of achievement is expressed by “aha moments” and partly in the need for an approach when puzzling over mathematical problems which can be found in the corresponding interview excerpts above.

The perspective of sense of achievement and perceived competence gives a possible explanation why so many students find the schema aspect intrinsically motivating: With predetermined procedures for predefined solutions sense of achievement becomes predictable, plannable, and reproducible. The following interview excerpt illustrates this close relation of intrinsic motivation and sense of achievement with a focus on schema aspect, indicated here for example by “I like calculating”:

T: I’ve always liked doing it [mathematics]. As I said, only when I understood it. Because when I don’t understand it, I’m always very frustrated and then I get a bit angry, and then I don’t like it anymore. But as soon as I understand it again, I enjoy it again. […] when I know I can do it and I’m calculating the tasks without having any big problems, then I enjoy it, I like calculating. Yes, and that’s how I ended up in the maths teaching programme. I just liked doing it [mathematics].

For the formalism aspect sense of achievement can mean to recognise the “wonderfully beautifully organised and clear structure” or the logic, unambiguosity, and precision of mathematics and thus experiencing mathematics as logical, comprehensible, and approachable for oneself:

T: I like its [mathematics] logic. If something is logical, then I find it nice. […] when everything is so clear and you can somehow handle it really well in your head, even though it is so abstract, then I find that somehow a good feeling. I mean when I understand it.

Especially the last sentence here (“I mean when I understand it”) indicates the significance of sense of achievement and perceived competence: Sense of achievement or perceived competence seem to be a prerequisite for mathematics-specific intrinsic motivation which, building on this, is then further differentiated in terms of the four different aspects of the mathematical worldview.

**Discussion**

The present paper gives a qualitative insight into students’ mathematics-specific intrinsic motivation for choosing mathematics as a study subject. The four aspects (process, application, formalism, schema) of the mathematical worldview according to Grigutsch & Törner (2002) turned out to be a suitable theoretical framework for qualitatively describing and categorizing different characteristics of
mathematics the mathematics-specific intrinsic motivation of the interviewed students referred to. Especially the framework given in Figure 1, that regards process and schema aspect as well as application and formalism aspect as opposite poles of a respective dimension was helpful to identify students’ positioning towards these four aspects.

Regarding Figure 1, it is noticeable how many preservice teachers enjoyed the schema aspect of mathematics. Process and formalism aspects were represented rather by the interviewed mathematics majors and rather rarely by the interviewed preservice teachers. The formalism aspect occurred here in two forms. The first form was mainly represented by two mathematics majors and identified the beauty of mathematics within its closed and consistent clear structure. The second form of the formalism aspect emphasised mathematics as being “logic” (for them), with unambiguous right and wrong, which is connected to the schema aspect. Considering that usual descriptions of university mathematics as being abstract, formal, and proof-based (e.g., Gueudet & Thomas, 2020) rather emphasise the formalism aspect, and especially its first form described above, such findings can contribute to explaining frustration (Göller & Gildehaus, 2021), demotivation, and dissatisfaction (Gildehaus & Liebendörfer, 2021a; Kosiol et al., 2019) of students who enjoy the schema aspect. In line with previous research, the results of this study indicate that this applies especially to mathematics preservice teachers (Gildehaus & Liebendörfer, 2021a; Göller & Gildehaus, 2021; Ufer et al., 2017). This means that for many preservice teachers, the aspects they liked about mathematics in school and because of which they have chosen mathematics as a study subject are unlikely to be recognised in university mathematics. Consequently, for a positive appraisal of university mathematics, many students have to develop new mathematics-specific interests i.e., adapt their mathematical worldview to which their mathematics-specific intrinsic motivation refers.

The here found significance of sense of achievement and perceived competence for students’ (intrinsic) motivation is well represented in common motivation theories. For example, in terms of expectancy-value theory (Eccles & Wigfield, 2002) it is represented in students’ expectancy (how well they will do in mathematics); in terms of self-determination theory (Ryan & Deci, 2000) it is represented in students’ perceived competence. Regarding the transition from school to university in mathematics, this significance of sense of achievement is likely to additionally complicate the situation. In the present study, sense of achievement seemed to be often closely linked with schema aspect (e.g., knowing what or how to calculate) or the second form of the formalism aspect described above (e.g., experiencing mathematics as being “logical” to oneself). Given the usual descriptions of university mathematics as well as the well-documented difficulties of students in the transition from school to university (e.g., Gueudet & Thomas, 2020), it can be expected that moments associated with such a sense of achievement will rather decrease. Accordingly, when designing university mathematics courses students’ mathematics-specific intrinsic motivation as well as the importance of sense of achievement should be taken into account in order to support students’ mathematics learning on as many levels as possible.

When considering civic education through mathematics, a dynamic mathematical worldview (i.e. an emphasis on the process and application aspect) seems rather fitting for implementing (normative) mathematical modelling (Gildehaus & Liebendörfer, 2021b; Pohlkamp & Heitzer, 2021; Steffensen et al., 2021) or critical mathematics education (Skovsmose, 2020; Steflitsch, 2021) in (future) classrooms. Accordingly, for a positive appraisal of such a way of mathematics education, many preservice teachers (especially those with a rather schematic mathematical worldview at the beginning of their studies) have to adapt their mathematical worldview, to which their mathematics-specific intrinsic motivation refers, first towards university mathematics and later towards a dynamic worldview for civic education through mathematics. In order to achieve a better compatibility, united efforts in the different educational institutions are necessary to build a broader mathematical worldview that supports and promotes several aspects of students’ mathematical worldview.

Finally, these results rise the question why the mathematics-specific intrinsic motivation of so many preservice teachers referred to the schema aspect. On the one hand, this question raises further ones about implicit teaching norms and practices in German schools. One the other hand, since the schema
aspect is less pronounced for mathematics majors’ intrinsic motivation, it should be discussed critically and further investigated, why so many students with a schematic worldview choose a teaching degree programme and what that means for later teaching.

When interpreting the results, the small qualitative sample of students from only one university should be taken into account. Therefore, further studies are desirable that examine mathematics-specific aspects of students’ intrinsic motivation and their relevance for students’ further academic studies and later profession in more detail.

References


