

CONVEYING DIFFERENT TYPES OF VALUES VIA MATHEMATICAL TASKS

TRANSMITIR DIFERENTES TIPOS DE VALORES A TRAVÉS DE TAREAS MATEMÁTICAS

*TRANSMITIR DIFERENTES TIPOS DE VALORES POR MEIO DE TAREFAS
MATEMÁTICAS*

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Recibido: 05/07/2023

Aprobado: 05/07/2023

ABSTRACT

The purpose of this paper is to analyse possibilities of including and conveying a variety of values in teaching mathematics through tasks in terms of their openness. In the first part of the introduction we present the theoretical ideas about values in mathematics education by Bishop and Lim & Ernest. The second part of introduction sets out the reasons why an emphasis on values seems advisable. Mathematics is not commonly associated with a variety of values. However, for mathematical education an orientation towards societal demands appear relevant. Scholars like Skovsmose and Fischer alike demanded the development of a reflecting ability. Some national state curricula depict some of the societal demands by calling for an orientation towards general values for all school subjects, but in this we see discrepancies to practiced teaching. The section “Tasks and Values” relates to justifying why tasks appear to be a useful means of influencing mathematics education. Several scholars have already investigated the possibilities of different task-types for teaching values. Here we connect this purpose to tasks characterized by their varying openness. As teachers are essential to unfold the potential of tasks, the present analysis is intended to give them concrete ideas for mediating values in mathematics classes. In the main part we analyse traditional tasks from ancient textbooks, with experiences in Armenian classes, as well as an open task with experiences of Austrian students in partner work. The discussed tasks provide opportunities for mathematical values like control, openness and for epistemological values as well as for more general values like self-confidence, mutual respect, justice and mathematics educational values like accuracy and consistency. In conclusion, the encouragement towards discussions among and with the students during the solving process of the task appears to be essential. Our experiences show that students mainly react positively towards such discussing activities. This could be a motivation for teachers to take up this issue besides of idealistic reasons.

Keywords: mathematics teaching. tasks. social values. epistemology. mathematics education.

RESUMO

O objetivo deste artigo é analisar as possibilidades de incluir e transmitir uma variedade de valores no ensino da matemática por meio de tarefas em termos de sua abertura. Na primeira parte da introdução, apresentamos as ideias teóricas sobre valores na educação matemática de Bishop e Lim & Ernest. A segunda parte da introdução apresenta os motivos pelos quais a ênfase nos valores parece aconselhável. A matemática não é comumente associada a uma variedade de valores. Entretanto, para a educação matemática, uma orientação para as exigências da sociedade parece ser relevante. Estudiosos como Skovsmose e Fischer exigiram o desenvolvimento de uma capacidade de reflexão. Alguns currículos estaduais nacionais retratam algumas das exigências da sociedade, ao exigir uma orientação para valores gerais para todas as disciplinas escolares, mas nisto vemos discrepâncias no ensino praticado. A seção "Tarefas e Valores", está relacionada com a justificativa de o porquê de as tarefas parecem ser um meio útil de influenciar a educação matemática. Vários estudiosos já investigaram as possibilidades de diferentes tipos de tarefas para o ensino de valores. Aqui conectamos este propósito às tarefas caracterizadas por seus vários modos de abertura. Como os professores são essenciais para desdobrar o potencial das tarefas, a presente análise pretende dar-lhes ideias concretas para a mediação de valores nas aulas de matemática. Na parte principal, analisamos tarefas tradicionais de livros antigos, com experiências em aulas na Armênia, bem como uma tarefa aberta com experiências de estudantes austríacos em trabalhos em parceria. As tarefas fornecem oportunidades para valores matemáticos como controle, abertura e para valores epistemológicos, bem como para valores mais gerais como autoconfiança, respeito mútuo, justiça e valores educacionais matemáticos como precisão e consistência. Em conclusão, o incentivo para discussões entre e com os alunos durante o processo de resolução da tarefa parece ser essencial. Nossas experiências mostram que os alunos reagem principalmente positivamente a tais atividades de discussão. Isto poderia ser uma motivação para que os professores assumam esta missão para além de razões idealistas.

Palavras-chave: ensino de matemática. tarefas. valores sociais. epistemologia. educação matemática.

RESUMEN

El objetivo de este artículo es analizar las posibilidades de incluir y transmitir diversos valores en la enseñanza de las matemáticas a través de tareas en función de su apertura. En la primera parte de la introducción presentamos las ideas teóricas sobre los valores en la enseñanza de las matemáticas de Bishop y Lim & Ernest. En la segunda parte de la introducción se exponen las razones por las que parece aconsejable hacer hincapié en los valores. Las matemáticas no suelen asociarse a una variedad de valores. Sin embargo, para la educación matemática parece pertinente una orientación hacia las demandas de la sociedad. Estudiosos como Skovsmose y Fischer exigían el desarrollo de una capacidad de reflexión. Algunos currículos estatales nacionales describen algunas de las demandas sociales pidiendo una orientación hacia valores generales para todas las asignaturas escolares, pero en esto vemos discrepancias con la enseñanza practicada. La sección "Tareas y valores" se refiere a la justificación de por qué las tareas parecen ser un medio útil para influir en la educación matemática. Varios estudiosos ya han investigado las posibilidades de distintos tipos de tareas para la enseñanza de valores. Aquí relacionamos este propósito con tareas caracterizadas por su diferente apertura. Dado que los profesores son esenciales para desplegar el potencial de las tareas, el presente análisis pretende darles ideas concretas para mediar los valores en las clases de matemáticas. En la parte principal analizamos tareas tradicionales de libros de texto antiguos, con experiencias en clases armenias, así como una tarea abierta con experiencias de alumnos austríacos en el trabajo en pareja. Las tareas analizadas proporcionan oportunidades para valores matemáticos como el control, la apertura y para valores epistemológicos, así como para valores más generales como la

confianza en uno mismo, el respeto mutuo, la justicia y valores educativos matemáticos como la precisión y la coherencia. En conclusión, el fomento de los debates entre y con los alumnos durante el proceso de resolución de la tarea parece ser esencial. Nuestras experiencias demuestran que los alumnos reaccionan positivamente ante estas actividades de debate. Esto podría ser una motivación para que los profesores se ocupen de este tema, además de por razones idealistas.

Palabras clave: enseñanza de las matemáticas. tareas. valores sociales. epistemología. educación matemática.

Introduction

In this paper our considerations are based on the definitions of values according to Bishop and Lim & Ernest. Bishop (2001) stated that “values in mathematics education are the deep affective qualities that education fosters through the school subject of mathematics” (p. 94). He categorized the values in the mathematics classroom into three groups: the general educational, the mathematical, and the mathematics educational. General educational values are not mathematical in nature and they “often have moral overtone and are essential for the maintenance and enhancement of the social fabric” (Bishop et al., 1999, p. 7). Mathematical values have been identified into three complementary pairs: rationalism and objectism, control and progress, openness and mystery (Bishop et al., 1999). Finally, the third group of values in the mathematics classroom reflect the values, which are both mathematical and educational (Bishop et al., 1999). Lim & Ernest (1997, p. 39) group the values into three categories:

- (i) Epistemological values – values involved with the acquisition, assessment and characteristics of mathematical knowledge and in epistemological aspects of the processes of teaching and learning mathematics, such as accuracy, systematicity and rationality.
- (ii) Social and cultural values – values which favour or support the social group or society and which concern the individual’s duty to society as related to mathematics education. Examples from this category are co-operation, justice and appreciation of the beauty of mathematics.
- (iii) Personal values – values affecting the individual as a learner and as a person, such as patience, confidence and creativity.

Values are often linked to a specific culture in which these values are acted. Mathematics as a science and practice is not commonly associated with a variety of values. In the Britannica dictionary we can find a general description of how mathematics is conceived, and according to that mathematics is dealing with logical reasoning and quantitative calculation and involving an increasing degree of idealization and abstraction in its development (Folkerts et al., 2022). This is in line with the view that mathematics is particularly connected to epistemic and aesthetic values (Skovsmose, 2020b).

As for school subjects in general, for mathematics education an orientation toward scientific practice is common. These approaches are criticized (rightly in our view) in the scientific debate regarding (mathematical) education focusing on the development of our society. Fischer (2012) and Skovsmose (2020a) elaborated their critique regarding their demand for reflective competence: Fischer (2012) considered the role of school education not in training specialists, but primarily in educating members of our society who are able to communicate, make decisions and judgements, and perhaps also to be critical. He stated that the approach to let the learners “do a little bit of what experts do” (p. 14) would not contribute to developing required reflection skills. Skovsmose (2020a) explicitly rejected mathematical activity as practiced at universities as a role model for mathematics education because in mathematical philosophy he sees a vacuum towards ethical issues. This would provide a too narrow scope for reflection on the effects of mathematics, caused by focusing on intrinsic quality criteria and neglecting the context in which mathematics operates. This might limit mathematics education as providing too little potential for reflection.

Similarly, an orientation towards values in mathematics education cannot be justified by an orientation towards values inherent in mathematics. This is consistent with various national states curricula, which prescribe the teaching of values. For example, the curriculum for general secondary schools in Austria

mentions “acceptance, respect, mutual esteem, and the ability to discourse”, but also the “development of self-confidence” and the “assumption of social responsibility” (Bundesministerium für Bildung, Wissenschaft und Forschung, 2023, p. 8, translated by CP). These kinds of values can be considered as general values in Bishop’s terms as well as social and cultural or personal values according to Lim and Ernest given above.

From our experience, we know that mathematics teachers have difficulties in teaching values, which they hardly can associate with mathematics in their lessons. For this reason, we would like to discuss here some tasks with regard to their potential to convey values, taking into account their different levels of openness. Selected student work is also included to provide a deeper insight into practical realisations. This exemplary elaboration should also serve as a proposal to the discussion in the MES community.

Tasks and values

In this article, we focus on mathematical tasks, as working on tasks is a central part of mathematics education. The importance of tasks for providing learning opportunities in mathematics has been demonstrated in various studies (see, for example, Sullivan et al., 2013, p. 14). Several scholars see tasks and their solving process as a way to educate values in the teaching process of mathematics. Taplin (1998) suggests problem solving as an approach to educate for human values, as it allows enhancing logical reasoning, being able to apply rules appropriately, or to create own rules, experiencing the full range of emotions associated with various stages of the solution process. In another paper she describes the types of problems, whose solving can enhance human values, and gives some suggestions of how it can be used in the mathematics program (Taplin, 2020).

Teaching values via the problem-solving approach in mathematics has been discussed also by Algani (2019), who connects solving of tasks with enhancing educational values such as friendship and cooperation, responsibility and self-reliance, innovation and social-engagement.

Daher (2020) discusses how to encourage the prevalence of moral and democratic values in the mathematics classroom and represents the results of his research, which indicate that values and democratic practices in mathematics classroom could be cultivated through taking care of the mathematical activity which needs to be rather open and encourage multiple solutions.

Mikaelian (2011, 2018) discusses the connection of values in general, and moral values in particular, with mathematics education. He shows that mathematics education has a great potential for the formation of values. Besides, in his textbooks of algebra he paid attention on the choice of applied background, anchoring it completely on national and universal values.

Consequently, one of the effective ways for the formation of values in the teaching process of mathematics is the inclusion of mathematical tasks which in their contexts include manifestations of specific values, are real-life problems, or are open tasks. Such tasks enable discussions between students, and teachers and students, they help to promote for developing students’ abilities, values, beliefs and attitudes (see Yenokyan, 2019). The purpose of this paper is to analyse the opportunities for conveying values in relation to the openness of a task. Open tasks give students room for decision-making, as the product that is created in the processing of the task is not exactly given. With this definition we are in line with Pehkonen (2017, p.10).

In the literature, open tasks are often linked to desired mathematics learning. The opportunities of such tasks can be found in their potential to provide challenges for all students (see Sullivan, 2013, p. 137). In providing the freedom of different approaches, students might more easily find their own starting points for the task and work on them. Different approaches provide different answers, and in their differences, they can still be judged as appropriate. Finally, the collection of the different results provides the whole class with the opportunity to get to know several approaches, as well as to compare, examine, defend and clarify them, in that way further developing their own knowledge. Nevertheless, these

opportunities are not necessarily fulfilled. It must be emphasised that the teacher's role and knowledge are essential to unfold the potential of tasks in the classroom, together with the students (ibid, pp. 15–18). Especially when students have little or no experience with open tasks, they may be overwhelmed by the range of possible starting points or they do not see the range of possible answers. In such situations the teacher is required to provide enabling or extending prompts. The collection of the results is also demanding for teachers as specialised pedagogical content knowledge is necessary.

In the context of teaching values in mathematics classes, we want to consider the opportunities but also the constraints of open tasks. A certain openness of tasks can provide opportunities for several values discussed by Bishop et al. (1999) and Lim & Ernest (1997). In this paper we would like to concentrate on tasks as examples with regard to the teaching of some of these values in mathematics lessons.

Examples of tasks

The first three of the following examples were produced in Armenian schools during an experimental lesson. The fourth example was carried out in interviews with Austrian students in partner work. These two different perspectives have in common that they give the opportunity to open discussions in mathematics lessons and so offering to the students to share their attitudes. Additionally, the different tasks provide the chance to elaborate on various forms of openness of tasks.

The first task is a “closed” task from an ancient Armenian textbook of Papikean (1875, p. 152, task 49, translated by AY), published in Venice:

A rich man bequeathed $\frac{1}{100}$ th of what he had to the church for renovation, 200 liras [note: former Italian currency] less than that to the school, and 200 liras less than the money he left to the school he bequeathed to the hospital. After giving these commandments, the heirs were left with $\frac{39}{40}$ of the property. How much lira was the property?

Usually teachers treat tasks like this as a closed task, bringing it to the classroom when students already know how to set up and solve equations. When teachers encourage students making explanations about their solving method and justifying their answers, it could be ground for the formation of mathematical values of control and openness mentioned by Bishop et al. (1999). When teachers encourage students to write their solutions accurately and being consistent during the solving process, could promote for the formation of mathematics educational values such as accuracy and consistency mentioned by Seah et al. (2001). Besides of using only the numbers mentioned in the task, setting up the appropriate equation and solving it, teachers could treat this task more openly by taking into consideration the context on two different levels. First by referring to the anciency of the tasks: a historicization (cf. Mikaelian & Yenokyan, 2021) could enhance the interest of students as well as refer to Armenian heritage. Second, by encouraging a discussion about the values of kindness, duty, gratitude and democracy. Additionally, questions like “What would you do instead of this rich man? How would you share the money and why?” could contribute to the openness of the task. In this way, all students regardless of their mathematical knowledge can participate and express their opinions about their choices. When discussing general values such as kindness, duty, gratitude and democracy, we consider a differentiated attainment: one could designate the man as kind because of providing some part of his money for welfare. On the other hand, one could refer to an actual discussion in Austria about inheritance taxes, taking the position that what happens with a big amount of money should be democratized (see Nimmervoll, 2021). Eventually, such discussions could contribute to recognizing mathematics as a collection of tools to execute calculations, but we decide which calculations to choose according to our value system.

The next examples refer to arithmetic. Their solving processes could be helpful for the discussion and for the formation of the value of justice. J. Rawls (1999), in his *Theory of Justice*, discusses equalizing and distributive principles of justice. The following tasks show how these principles can be addressed. The first one is taken from Aghamaleants (1781, p. 196, translated by AY) and refers to the distribution of benefit in an economic context.

Petros, Poghos and Hovhannes made friends in business and invested a certain amount of money together. Petros invested 1000 dahekan [note: ancient currency], Poghos 2000 dahekan and Hovhannes 3000 dahekan. They got 1800 dahekan as a benefit and wanted to share among each other. Find out how much dahekan each one has to get.

This task was worked on by students in an experimental lesson in the context of Yenokyan's (2019) doctoral thesis. At first, most of the students (aged 12 to 13) said "we have to divide 180 to 3, so each one will get 60 dahekan". However, after highlighting the different amounts of investments, they came up with the idea to apply proportionality. However, during the solving process, they had the opportunity to discuss these two options within the given context, including argumentations of the two principles of justice. For promoting students' skills of solving this kind of tasks, the following variation of the task was offered:

Armen, Arman and Artur [note: the names were chosen instead of A, B, C] made friends in business and invested a certain amount of money together. Armen invested 24000 dahekan, Arman 30000 dahekan and Artur 36000 dahekan. They got 3000 dahekan as a loss and wanted to share among each other. Find out how much loss of dahekan each one has to get. (Papikyan. 1863, p. 466, translated by AY)

Expecting, that all students would choose the proportionality, because of the similarity of the two tasks, Narek (all names changed) used the chance to open the task in regard of his moral considerations. He argued to use equalizing justice, so each person would have the loss of 1000 dahekan. Being asked for justification, he responded:

Yes, in the case of previous task and also, in this case it will be MATHEMATICALLY correct, if we use distributive principle of justice [note AY: he referred to proportionality], but I don't want to choose it, because if I were Armen, I would offer to divide the loss equally, because when you lose something in your business it is very difficult and terrible, and if we are true friends we have to stand by each other and support each other during difficult situations. This is a true friendship for me. (Translated by AY)

Narek's statement gave the opportunity to discuss the value of friendship in class plenary. This shows a possibility to redesign the question of the task and make it more open, for example "Which option would you choose to divide the loss and why?".

Friendship is also an issue of the next task, titled "Facebook friends" (Plunger, in press). It takes up the content area of descriptive statistics:

In order to indicate a representative, characteristic value of a data series, measures of centre such as the arithmetic mean, the median, or the mode are used in descriptive statistics (among others). For a study, a suitable mean for the number of "Facebook friends" of all Austrian Facebook users shall be specified. Explain which of the above measures of centre you find most suitable for this purpose.

The aim of the task is to give arguments for a choice that are comprehensible and mathematically correct in the given context of Facebook friends. The task is open in regard of the answer, as students are free to choose one or more of given measures of centre, although the choice not to use any of the given measures is not really given. In terms of the method, the task is also open: the argumentations can refer to reasons for excluding a central measure, or to why a chosen one is particularly suitable. It doesn't matter whether the learners first deal with the mathematical concept or the described context, or immediately relate both to each other. However, the considerations should include both mathematical (e.g., related to the data type) and contextual aspects (e.g., the meaning or interpretation of a central measure in the context).

Pairs of students from the 8th grade (aged 14 to 15) worked on this task during clinical interviews, conducted in the course of a research project for investigating reflection processes (Plunger, 2021). The Facebook friends' task was supposed to encourage model-oriented reflection, a type of reflection which focuses on thinking about mathematical models and their fit, limits, effects and implicit assumptions for the extra mathematical situation (Schneider, 2019, p. 4).

In the context of values, this task seems to be suitable to promote self-confidence: at the time students were confronted with this task, they had already dealt with the measures of central tendency. Hence, they had strategies to examine at least one of the concepts, and even if they had forgotten about one of these concepts (in this case the partner or interviewer could easily help) or mixed up terminology, they still felt competent about relating the concept to the context of Facebook friends. Among the students, Facebook was probably one of the less trendy social platforms, but they all seemed to have enough experience with the concept of friends on Facebook to feel confident with this context. Familiarity with Facebook should be considered if this task is used in future years. The question of the task alone picks up on the fact that there is not a one and only true formula for determining a suitable representative, characteristic value. Rather, it demonstrates that mathematics provides various concepts from which to select the appropriate ones. It is left to the students to judge which criteria should be considered. The interviewed pairs used a variety of different aspects and related them to each other, such as sensitivity to outliers, the familiarity of the concepts, the presumed distribution of friends, etc. Despite the openness of the task, the results of the interviewed pairs can be assessed in relation to the purpose of the task. The pairs were able to come up with suitable arguments, but also deficits in content knowledge became apparent, and there were some possible starting points for deeper mathematical considerations. A more detailed analysis of the interviewees' content-related ideas is provided elsewhere (Plunger, in press). In a classroom situation it would be necessary to discuss the results of the students, and self-confidence could be diminished, if students realise that some of their findings can be relativised or aspects of their solutions were wrong. Still students will unlikely produce only mathematically wrong procedures or unreasonable arguments, as there is a variety of possibilities.

In the course of the interviews, the students did not comment on what meaning of friendship is captured by such a representative value at all, or to what extent such a frame of friendship covers their own ideas of friendship. Nicolo may have touched on this level: "for me, all three of them make pretty little sense in something like this. Honestly." Here, the framework of the interviews was obviously not open enough to describe in more detail the inappropriateness felt by this student. He and his partner nevertheless aimed to find suitable arguments for the single measures of centre. A variation of this task including real numbers of Facebook-friends of a group of people and the question to determine the single measures of centre and to compare them with each other and to the students' own conception of friendship could make this task more closed, but provide a certain openness in giving an occasion to talk about the concept of friendship within mathematics lessons.

Interview excerpts from two other pairs of students show how working in pairs on this open task can help to practice cooperation and making sense, in two quite varied ways. The passage from Klara and Joelle reflects an excerpt from the beginning of their conversation about the task. It follows Joelle's argument for the median, as the arithmetic mean does not seem appropriate because the result can be a decimal number, and Klara's attempts to still consider the arithmetic mean (the result can be rounded, it seems to be more precise) (translated by CP):

- 69 Klara: So, I actually find your thought quite clever, that there are no half people
[laugh]
- 71 Joelle: [laugh], yes.
- 72 Klara: But that was often the case with us in class.
- 73 Joelle: I mean, yes. We could. I don't know, no actually now it does not matter, I would just say, we could have a look, so if one had ten friends, one could calculate in mental arithmetic how it would be with BOTH ways. Because then you would count and you would get five for the median and for the-
- 77 Klara: -arithmetic mean would not work at all.
- 78 Joelle: Sure, all the numbers together and divided by ten.
- 79 Klara: Then the mean would be one
- 80 Joelle: No, since you have one plus two is three

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- 81 Klara: Huh?
- 82 Joelle: Arithmetic mean means ALL numbers are divided by the total number of numbers. Yes, and then you have ONE plus two plus three plus four plus five plus six up to ten and then by ten.
- 84 Klara: Right.
- 85 Joelle: [laugh]

In line 69 we see that Klara shows Joelle that she respects her reasoning. Still, by referring to the experiences in class (line 72), she shows that Klara's argumentation may be too short-sighted. Klara seems to accept this, and begins to consider further strategies for dealing with the task (from line 73, looking at concrete examples and determine the arithmetic mean and the median). Apart from the fact that there are indications that the two are not sure about the data from which the measures of centre are to be calculated, we see that Joelle corrects Klara's result and explains how the arithmetic mean is being applied here in her eyes.

In an interview excerpt of Gina and Hannah we see a mathematically incomprehensible argumentation of Hannah for the exclusion of the median.

- 17 Hannah: Well, I do NOT use the median, because the range would certainly be large, since if one somehow has only one Facebook friend and one somehow seven hundred or so, then the median would not be (...) There are MANY people on Facebook, Gina!
- 21 Gina: Yeah yeah, go ahead. Especially say why not for this reason.
- 22 Hannah: Well, because then the median wouldn't, I think the median would have a quite big difference to the arithmetic mean.
- 24 Gina: Yes, but the median is not influenced by outliers, not so directly, mh, not at all. And the arithmetic mean is. [Laugh]
- 26 Hannah: Ok. Yeah, ok that's right.

Gina and Hannah, like Klara and Joelle, are very collegial with each other. This seems to be helpful in recognising and accepting that a thought may need to be discarded again. However, Gina asks Hannah to explain in more detail how this should be a reason to exclude the median (line 21). She adds her argument for the median (and against the arithmetic mean) to Hannah's further explanation. We see how the two defend their own positions and also that Gina is not satisfied with Hannah's reason given in lines 17–19, but points out exactly what is not plausible to her. This passage is followed by a discussion of the mode, after which the two return to Hannah's argument and a more detailed discussion of it leads them to reject it:

- 66 Hannah: Um, I just wrote since the range is certainly very large and so the median would be far from the minimum or the maximum value. Does that make sense?
- 68 Gina: Yes, but that would be always the case, wouldn't it?
- 69 Hannah: Yes, it does (...). Ok, that would be a bad argumentation? Mh?
- 70 Gina: [Laugh]

The difference between the two pairs is, that Gina tries to find out why Hannah is coming to her argument, while Joelle does not give the possibility to Klara to explain her procedure. We could consider the situations of both pairs as a contribution of control in Bishop's (2001, p. 103) terms, as they are checking reasons for the answers being right or not. Still, we see that the in-depth discussion of Hannah's considerations led to plausible results. In Klara and Joelle's case Joelle was not interested in Klara's reasonings. Of course, we don't know where Klara's explanations would have led, but there might have been the chance to work on the question which data the calculations should be based on.

Conclusion

The presented examples show how mathematical tasks can contribute to convey values by their nature of openness. It can be seen that a certain openness in the perception of a task is needed. Still, we should be aware that frameworks in which students work on the task are important like Narek and Nicolo show, when considering the concept of friendship. In Nicolo's case nor his partner, nor the interviewer did take up the felt inappropriateness: this could indicate that the situation was too closed for this to happen. On the other hand, Narek's unexpected solution approach and view of friendship made the classroom-situation more open and were a base for further discussions. Additionally, with the analysed examples we see that tasks may refer to different mathematical contents, extra-mathematical contexts or methods of working, while still having the potential to address several values discussed by Bishop et al. (1999) and Lim & Ernest (1997). To us it seems essential to encourage students for involving in discussions during the solving process. We are aware that including tasks with specific context and differing openness for explicitly focusing on values imply very high demands for teachers. They could be motivated to take up the mission of including the formation of values in mathematics lessons because of idealistic reasons. Besides, in our experiences with students, they appreciated this kind of discussions. This might also be due to the fact that even students who do not "shine" with mathematical knowledge are included in discussions and in this way, they feel more confident in the classroom. However, our experience also shows that some students do not always see these discussions at the core of mathematics education.

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