

SIMULATION-BASED MATHEMATICS AND SOCIAL JUSTICE ACTIVITIES

LEARNING FROM STUDENT REFLECTIONS

ACTIVIDADES DE JUSTICIA SOCIAL Y MATEMÁTICAS BASADAS EN SIMULACIÓN

Aprendiendo de las reflexiones de los estudiantes

ATIVIDADES DE MATEMÁTICA E JUSTIÇA SOCIAL EMBASADAS EM SIMULAÇÃO

Aprendendo com as reflexões dos alunos

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ABSTRACT

In this paper, various dimensions of care involved in mathematics education are linked to the need to develop classroom activities that connect mathematics and social justice issues. Drawing from literature that shows cognition is situated and embodied, the importance of meaningful contextualization and social interaction when learning mathematics is highlighted. The concept of “simulation-based mathematics and social justice activities” is presented as an approach for the work of bringing social justice issues that have mathematics at their core to the classroom. Theoretical constructs and examples are discussed, to illustrate what such simulations may entail and what may be learned from scholars, in different educational fields, who use simulations in classroom activities. Potentially beneficial features of social justice simulations are related to various educational goals, such as: decreasing arbitrary boundaries between mathematical sub-areas and between mathematics and other disciplines; providing opportunities for choice and the embodiment of different perspectives; and offering opportunities for inter-personal learning. I report on a simulation-based mathematics and social justice activity, conducted in a teacher education classroom. Students—in this case future teachers—were prompted to write reflectively about their participation in the activity. My interest lies in finding out what, if any, cognitive and affective benefits these prospective teachers connect with their experience of mathematics in the activity. To investigate this, I analyze six themes that are present in the data and illustrate each theme using excerpts of student writing. The thematic analysis allows us to learn about connections made by these students, between mathematics, in the context of the activity, and issues that are personally meaningful to them, such as: their own future teaching practice, learning and interacting with peers, beliefs and feelings about mathematics and the learning of mathematics, and different perspectives on complex decisions, involving cooperation or lack thereof, that are encountered in real life situations.

Keywords: simulations. mathematics and social justice.

RESUMEN

En este artículo, varias dimensiones de cuidado involucradas en la educación matemática están vinculadas a la necesidad de desarrollar actividades, para el aula, que conecten las matemáticas y cuestiones de justicia social. Utilizando literatura que muestra que la cognición está situada y corporeizada, se destaca la importancia de la contextualización significativa y de la interacción social al aprender matemáticas. El concepto de “actividades de matemáticas y justicia social basadas en simulación” se presenta como un enfoque para el trabajo de llevar al aula cuestiones de justicia social que tienen a las matemáticas en su médula. Se discuten construcciones teóricas y ejemplos para ilustrar en que consistirían tales simulaciones y lo que se puede aprender de los académicos, en diferentes campos de la educación, que usan simulaciones en actividades de aula. Características potencialmente beneficiosas de las simulaciones de justicia social son relacionadas a varios objetivos educativos, tales como: reducir fronteras arbitrarias entre subáreas matemáticas y entre las matemáticas y otras disciplinas; brindar oportunidades para hacer elecciones y incorporar diferentes perspectivas; y ofrecer oportunidades para el aprendizaje interpersonal. Presento una actividad de justicia social y matemáticas basada en simulación, realizada en un aula de formación docente. Se invitó a los estudiantes, en este caso futuros maestros, a escribir reflexivamente sobre su participación en la actividad. Mi interés radica en descubrir qué beneficios cognitivos y afectivos, si los hay, estos futuros maestros conectan con su experiencia de las matemáticas en la actividad. Para investigar esto, analizo seis temas que están presentes en los datos e ilustro cada tema usando extractos de los escritos de los estudiantes. El análisis temático nos permite conocer conexiones que hacen estos estudiantes, entre las matemáticas, en el contexto de la actividad, y cuestiones que son significativas personalmente para ellos, tales como: su propia práctica futura de enseñanza, aprendizaje e interacción con pares, creencias y sentimientos sobre las matemáticas y el aprendizaje de las matemáticas, y diferentes perspectivas sobre decisiones complejas, que implican cooperación o falta de ella, con las cuales nos enfrentamos en situaciones de la vida real.

Palabras clave: simulaciones. matemáticas y justicia social.

RESUMO

Neste artigo, várias dimensões de cuidado envolvidas na educação matemática são ligadas à necessidade de desenvolver atividades, para sala de aula, que envolvam temas de matemática e justiça social. A importância da contextualização significativa e da interação social no aprendizado de matemática é ressaltada utilizando literatura que mostra que a cognição é situada e corporificada. O conceito de “atividades de matemática e justiça social embasadas em simulação” é apresentado como uma abordagem para o trabalho de trazer à sala de aula temas de justiça social que têm matemática na sua medula. Construções teóricas e exemplos são discutidos, para ilustrar em que consistiriam tais simulações e o que podemos aprender de pesquisadores, de diversas áreas da educação, que usam simulações em atividades de sala de aula. Características potencialmente benéficas das simulações de justiça social são relacionadas a vários objetivos educacionais, tais como: reduzir fronteiras arbitrárias entre sub-áreas da matemática e entre a matemática e outras disciplinas; proporcionar oportunidades de escolha e a corporificação de perspectivas diferentes; e oferecer oportunidades para aprendizagem interpessoal. Aqui, eu apresento uma atividade de matemática e justiça social embasada em simulação, realizada numa aula de formação de professores. Os estudantes — neste caso futuros professores — foram convidados a escrever em reflexão sobre a sua participação na atividade. Meu interesse reside em descobrir quais benefícios cognitivos e afetivos, se houver, são relacionados por esses futuros professores à sua experiência de matemática na atividade. Para investigar isso, analiso seis temas presentes nas reflexões dos alunos, e ilustro cada tema com trechos das reflexões. A análise temática

nos permite aprender sobre conexões, feitas por estes alunos, entre matemática, no contexto da atividade, e questões que são pessoalmente significativas para eles, tais como sua própria futura prática de ensino, aprendizado e interação com colegas, crenças e sentimentos sobre matemática e sua aprendizagem, e diferentes perspectivas sobre decisões complexas, envolvendo cooperação ou sua falta, com as quais nos deparamos na vida real.

Palavras-chave: simulações. matemática e justiça social.

Introduction

Critical scholars such as O. Skovsmose (2011) see mathematics education not as neutral but as undetermined, in the sense that it can be “acted out in many different ways and come to serve a grand variety of social, political, and economic functions and interests” (p.2). Reflecting on my own practice as a mathematics teacher, I have found that for many years my actions were mostly consistent with mathematics education as a neutral activity, unrelated to social context, emotions, politics, or economic interests. This stance became increasingly untenable over time because it clashed with my need to care for students. It also clashed with my care for the learning of mathematics, as it became more and more difficult to accept that, for too many people, “mathematics is not something they can make sense of, but rather something almost completely arbitrary (or at least whose meaningfulness is inaccessible to them)” (Schoenfeld, 1992, p. 10). Finally, in also caring for social justice in the world, I became interested in the potential to meaningfully connect mathematics education with a better understanding of social issues of concern.

Theoretical Frameworks

Cognition as Situated and Embodied

Research that focuses on the social and contextual nature of knowledge and cognition has shown that learning mathematics is situated and context-dependent (e.g., Núñez et al., 1999). A classic example is Carraher et al.’s (1985) evidence of students’ mathematical ‘ability’ drastically changing when the context changes from the classroom to the market where they work. Students’ statements about school mathematics can vary, as Boaler (2002) shows, depending on the norms and relationships that constitute the classroom community of practice, from: “a strange and specialized type of code [used] in one place—the mathematics classroom” (p. 131), to: “it has the same atmosphere” (p. 134) as mathematics outside school. Watson (2021) draws on evolutionarily informed education approaches to put it very simply: “humankind develops skills and tools in meaningful environmental contexts. Therefore...developing tools in meaningful contexts is how our brains must work” (p. 33). Núñez et al. (1999) further remind us that “[c]ognition is embodied; it is biologically grounded in individuals who interact with each other” (p. 53). It is therefore to be hoped that, when students reflect on their mathematics classroom activities, themes that emerge include their social interactions, bodies beyond just intellect, their lived experiences, and what matters to them. Below, I introduce a new approach in mathematics classroom activities, and offer a preliminary investigation based on themes that emerge in students’ reflections about such an activity.

Mathematics and Social Justice as Another Dimension of Care

For Eric (Rico) Gutstein (2012a), care for mathematics, learning, and the learners of mathematics, is intertwined with an additional dimension: care for “the struggles of [students’] communities for justice” (p. 43). Gutstein (2012a) describes how his teaching practice involves the use of *generative themes* (Freire, 1970/2000, p. 96), that is, “key social contradictions in people’s lives” (p. 26). He explains that “together [we] decided which contexts to study..., either based on students’ generative themes that they proposed, or on themes I suggested that they accepted” (Gutstein, 2012a, p. 30). Gutstein’s research, drawing on Freire’s critical pedagogy, shows that, through the emergence, mathematical modelling, and

discussion of such themes, there is potential to generate both social justice and mathematics understanding.

In the process of eliciting generative themes, a teacher may initially work with students to broaden conceptions of mathematics by engaging in dialogue about what mathematics is, its roles in the world, and how it connects to social issues. An example is Skovsmose's (2011) discussion of 'mathematics in action', using projects that help students to "reflect *on* mathematics, reflect *with* mathematics, and reflect *through* mathematics" (p. 72). Furthermore, the group (of students and teacher) may not have shared lived experiences with a social justice theme, as well as the wherewithal to model it mathematically (see e.g. Jablonka & Gellert, 2012, or Leonard & Moore, 2014). My research aims to assist and investigate teachers and students in this work, by providing, for classroom use, various mathematics, and social justice (henceforth MSJ) *activities*, that is, lessons centred on students' active participation (Bishop, 1985), that contain *simulations*. The activities aim to bring to life social justice issues which have a "math engine" (Finkel, 2020), that is, that have mathematics at their core, or can be understood using mathematics. *Simulation-based MSJ activities* consist of: a brief introduction to set the scene; a role-play, in which students simulate a scenario of social concern; discussion, in which the group collectively shares how events played out and works to understand the mathematics underlying the situation; and finally, reflection, in which connections to students' own experiences, interests, and knowledge of the world may emerge.

Simulations in MSJ: Examples and Beneficial Features

What might be some theoretical constructs and examples from education, broadly speaking, that may guide us in constructing MSJ simulations? As it would not make sense to restrict the flow of diverse ideas that may better fit various MSJ concepts or educational contexts, the examples and proposed features italicized in this section are offered as inspiration rather than prescription.

To begin, Bernstein's (1973/2008) construct of 'classification', or "degree of boundary maintenance between contents" (p. 366), is useful to keep in mind when creating MSJ simulation-based activities. Internal classification refers to the sectioning of a subject's sub-areas; by lowering it, we allow more connections to be made between mathematical sub-areas. External classification refers to separations between a subject and other areas; by lowering it, we allow more connections to be made between mathematics and other disciplines or everyday practice. Thus, aiming for *low internal and external classification* reflects an intention to reduce the chance that arbitrary boundaries, between mathematics knowledge and knowledge from lived experience, culture, and other disciplines, are passed on to students. MSJ activities aim to broaden conceptions of mathematics, what it can be used for, what counts as mathematical experiences, and who we are when we practice mathematics.

Simulations have long been used in education, and particularly in learning about topics that include stepping into others' shoes, by helping students experience injustice in a safe, classroom-adapted way. An example that is not from mathematics education but from which much can be learned is Jane Elliott's 1968 simulation of discrimination based on physical characteristics in her 3rd grade classroom. Elliott wondered if students could learn how it feels to discriminate and to be discriminated against because of skin colour. She simulated this in her class, using eye colour: brown- (or more generally not blue-) eyed students were given privileges and told the additional melanin in their eyes meant they were smarter. We learn from Elliott that simulations do not have to mimic exactly what happens in real life, but that they must capture the spirit of the situation in a coherent way. Scholars in social sciences education (e.g., Chin et al., 2009; Asal & Kratoville, 2013) express this idea by saying that, to be effective, simulations must have *verisimilitude*. Elliott ran the simulation a second time, with reversed student roles (where blue-eyed students were privileged), so that students could experience both sides of the situation. The *embodiment of different perspectives* was thus made possible for students. Students wrote essays about what they learned, which were published in a local newspaper and sparked discussion in the whole community. Elliott reported: "Students said, 'I found out what it felt like to be on the bottom, and I did not want to make anyone feel like that ever again'" (Bland, 2018). We learn from Elliott's experiment

the importance of students experiencing more than one side of a situation in the development of *empathy for different perspectives*, and the authentic learning power of having a *real audience* that exists beyond the classroom (Herrington et al., 2014).

In the mathematics education literature, the word simulation is often connected to the use of technology. For example, Andrà et al. (2015) report on their study of students using an app they created to *generate awareness* of the injustice of an Italian lottery and of the dangers of gambling. The app simulates playing the lottery, while also tracking (and showing students) how much money is spent versus prize money earned. Students experienced the draw of the game as well as learning the mathematics that shows the unjust setup of the lottery, while researchers studied the simulator's effects on students' emotions and cognition. We learn from Andrà et al. (2015) that simulations can provide an *opportunity to collect local data*, based on students' decisions, actions, and inferences in their own social setting; mathematical tools can then be used to contrast local, classroom-generated data and inferences with data and related policy decisions from wider, or global, contexts.

My interest, however, lies in simulations that focus on the power of *group social physical interactions*, and I am not choosing to study technology-based simulations. I choose to focus on students' construction of reality (including mathematics) based on "culturally determined forms of sense-making which are ultimately grounded in bodily experience" (Núñez et al., 1999, p. 49).

Here I report on an MSJ activity centred on simulating a widely known thought experiment called the 'prisoner's dilemma' (henceforth, PD) and on discussing an accompanying model from game theory (Nash, 1950; Kuhn, 2019). As the data from student reflections will indicate, the simulation of a PD-type scenario can bring to the fore issues of cooperation, trust, systems that induce the lack thereof, individual versus collective thinking, and mathematical rationality. These are universal issues of concern in the world, impacted as it is by a lack of cooperation in global crises such as wars and climate change. I suggest that *universality*, as a feature of simulated social issues, strengthens the foundation for "shared meanings" (Bishop, 1985), including mathematical meanings, to emerge. The activity also features *choice*, which, as Finkel's (2020) work with mathematical games shows, opens up learning opportunities as students ask strategy questions (such as what is the best choice to make or why someone else chose differently), and tends to hold students' interest longer than activities where choice is not involved or that stay within the right/wrong dichotomy often associated with mathematics. Moreover, this activity is *extendable*, as the concept of Nash equilibria in PD-type games can be used to illuminate other social justice issues, such as weaknesses in plurality voting systems (Gelman, 2003).

As Gutstein (2012b) points out, "one important issue in teaching mathematics for social justice [is] engaging students" (p. 64) who might otherwise be uninterested or apathetic. Besides engagement, social science education scholars have shown that classroom simulations which reflect the real world can provide learners with a variety of cognitive and affective gains in: subject-matter interest, knowledge, and skills; communication abilities and connections with peers; appreciation for complex decision-making; and empathy in real-life situations (e.g., Greenblat, 1973; Jones & Bursens, 2015; Szafran & Mandolini, 1980). In mathematics education, it seems plausible that we might find some of the same gains, but this has not yet been shown. Scholars such as Borasi & Rose (1989) have shown that evidence of student growth in areas such as "affective issues, the learning of mathematical content, the process of doing mathematics, and mathematics beliefs" (p. 353) can be gleaned from students' reflective writing about classroom activities. For example, students may show, through their writing, changes in their own interest for or anxiety towards mathematics. Here, I examine students' reflections to study: what, if any, cognitive and affective gains may be connected to students' experience of mathematics in the context of a simulation-based MSJ activity?

Methodology

Context and Description of Activity

The context for this study is a Simon Fraser University, Canada, teacher education course called ‘Struggle with Mathematics: Sources and Recovery’, which is designed for future secondary teachers who do not have a mathematics or science specialization. The 2-hour activity took place during one of the course’s regular classes, and presented the ‘prisoner’s dilemma’ (Kuhn, 2019; Nash, 1950) as follows:

Two criminals are arrested together and imprisoned. Each prisoner is in a separate cell with no means of communicating with the other. The police do not have enough evidence to convict the pair on the main charge of robbery, and plan to sentence both prisoners to a year in prison for weapon possession. But the police separately offer each prisoner a bargain: to go free if they will give evidence against their partner. This is the well-known prisoners’ dilemma. The possible outcomes are:

- If A and B each betray the other, each serves two years in prison;
- If A betrays B but B remains silent, A is set free and B serves three years in prison;
- If A remains silent but B betrays A, A serves three years in prison and B is set free;
- If A and B both remain silent, both serve one year in prison (on the weapons charge).

This scenario was then simulated, with the 34 students walking around the classroom, choosing different partners with whom to play the role of the two prisoners. Each role-play consisted of the partner pairs making independent simultaneous decisions —as in the game theory concept of simultaneous games, where each player chooses their action without knowledge of the other player’s choice of action—to cooperate or defect. Students noted the consequences of their choices, that is, how many years of prison each one got, before moving on to another partner. Students created a diagram to show the possible outcomes of the PD (see, for example, Sam’s diagram in Figure 1 below), and kept track of results from running the simulation several times. This was followed by a discussion in which the experience of the students was translated into mathematics. Most students pointed out a tendency to defect, that is, not cooperate with the fellow player. Others pointed out the advantage of mutual cooperation. The dilemma, experienced during the simulation, is that the defecting strategy, because it is dominant, tends to result in both prisoners defecting, instead of both cooperating to achieve a more beneficial outcome for both parties. The “notion of an equilibrium point” (Nash, 1950, p. 286), or Nash equilibrium, was introduced as the result of a thinking strategy that was deemed rational: that no matter what one’s partner chose, it was in one’s self-interest to defect. This supposed rationality was put into question by looking at examples such as price wars between companies that sell a comparable product. From there, various students offered their own examples, ranging from sibling rivalry to ‘tragedy of the commons’ situations that can and do arise in the sharing of (finite) natural resources. Applying the model to today’s lack of cooperation in global crises such as wars and climate change, students questioned the rationality of the entire system of (lack of) incentives to international cooperation.

Data Collection and Analysis

Here, I analyze the reflective writing of four PSTs (under pseudonyms) who chose to write about the MSJ activity based on simulation of a PD-type scenario, described above, as part of a course assignment, responding to the following prompt:

Reflect on your experience solving the problem, being in a group setting while solving the problem, what attracted you to the problem (or what was unpleasant about the problem). If the problem was too easy maybe you want to add an extension. Suggest where a student may have anxiety in answering your extension.

I began my analysis by seeking themes from Borasi & Rose’s (1989) study that were present in the data, across the writing of the four PSTs:

- learning of mathematics

- therapeutic value, (adapted here to focus mostly on mathematics anxiety)
- changes in conceptions of mathematics

More themes emerged from reading the data searching for topics repeatedly touched on by all the students. Zarestky & Bigler (2021) refer to the importance of such open coding, noting that if a “study’s purpose [is] to explore student thinking, open coding support[s] reliance on the data rather than any preconceptions of what students might or should write about” (p. 57). The additional themes are:

- connections between mathematics and real life
- perceptions about the peer interactions that were part of the activity
- learning as PSTs

As Zarestky & Bigler (2021) state, “[a]s might be expected, themes are highly interconnected” (p. 57). Indeed, student descriptions of their understanding of the PD model are not easy to extricate from other aspects of the activity, such as peer interaction and connections between the PD and real-life issues of cooperation, which made it difficult to completely avoid repetitions in excerpts across themes.

Theme: Learning of mathematics

Students reflect on mathematics in their writing: the PD model, Nash equilibrium, and the potential for probability as a lens to work with PD-type problems. Also, after the storied introduction and one round of PD playing, students were prompted to create their own visual representation of the PD.

Lucas: Lucas describes the dilemma as “[a] mutual assured destruction situation wherein the parties think that they are acting in their best interests, but then get stuck in a loop of diminishing returns” and the thought process for finding the Nash equilibrium as “pinpointing what scenario is most beneficial for a party regardless of the decision of the other.”

Sam: “I like how we started this exploration by trying to imagine what each scenario would result in and then had to make our own diagrams of the scenarios...here is mine:”

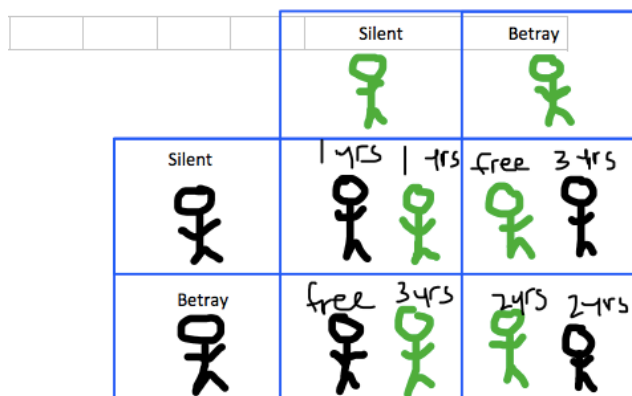


Figure 1: PD diagram by Sam

Jess: Jess sees probability at play in the activity: “I always found ‘odds’ interesting. (However, this activity wasn’t dictated by probability, but you also had to consider who your partner in the game was)... the level of trust puts an interesting spin on the experience and what ‘strategy’ is used, even though technically it was in your best interest to betray from the first turn.”

Theme: Therapeutic value and mathematics anxiety

Several associations are made by the students between the activity and feelings. Although there are suggestions for improvement related to consistency of language use during the activity as a whole, students otherwise report positively on the simulation as reducing mathematics anxiety. It is particularly important to learn from what PSTs write on this theme, since teachers can “pass their own anxiety, fear, and other negative qualities to their students” (Zarestky & Bigler, p. 62).

Lee: “I find when working with the same group members, we ...feel anxious exploring the questions and can project each other's anxieties onto other members of the group. I think mixing up the groups ...will allow for a more comfortable and safe environment to explore mathematics and ultimately will decrease math anxiety.”

Sam: “I don’t think this activity would bring up much anxiety because it is a fun activity, that you can’t really get wrong. The only anxieties I could see this bringing up would be social anxiety because you have to interact with other people.”

However, negative feelings are reported by both Sam (“feeling overwhelmed”) and Lee (“confusing”) due to changes in language, from ‘betray’ or ‘stay silent’, used in the storied introduction to PD, to ‘cooperate’ or ‘defect’, used for real-life examples. This reflection points out an instance of pedagogical discourse that means well but actually creates barriers for students. It is also an example of student reflections that lead to learning for myself as I work to improve future versions of the activity.

Lee: “The changes in wording during this exploration were unpleasant for me. I believe this could be a potential issue when dealing with students’ math anxieties.”

Theme: Changes in conceptions of mathematics

All the students report that the activity was fun, which is perhaps not surprising given that they chose this activity to reflect on. Students mention the absence of the right or wrong dichotomy often associated with mathematics, and the novel embodied experience of a simulation in math class:

Lee: “I did not realize that you could perform simulations in a math class, and thought that this class would be similar to my secondary math experience where we only answered math equations in a textbook... This activity showed me that math can be creative... [and] hands-on.”

Sam: “it is a fun activity, that you can’t really get wrong”

Lucas: “a lot of fun to... participate in and succumb to a particular mathematical phenomenon”

Theme: Connections between mathematics and real life

These reflections point to students' perceptions of verisimilitude and low external classification of the activity.

Lucas: Lucas describes the activity as “an experiment in rationality, and subsequently trust,” and adds: “[w]hat I did not expect was the activity’s evolution into an examination of human nature and capitalism, which to me renders this activity not only incredibly fascinating but also very accessible, especially to those who may not consider themselves strong math students but are very interested in social behaviours.”

Lee: “When applying this concept to examples in the economy like marketing strategies, I can understand why large companies cannot get out of the race to the bottom because they do not cooperate to receive a higher reward.”

Theme: Perceptions about the peer interactions that were part of the activity

Students connect the peer interactivity aspect of the simulation with learning opportunities (such as conversations about strategy and validation of different perspectives), experiential learning, more fun, and reduced mathematics anxiety.

Sam: “enjoyed... that we got to get out of our desk and put the scenario to the test with our classmates... [a peer] explained to me why it’s actually in my best interest to betray people. She told me that it is ideally better to betray people because if you betray your partner and your partner stays quiet you would walk free....I hadn’t really thought about a strategy that would benefit me before going into the scenario.”

Lucas: “The exemplar reaches its full potential in a group setting as we are directly engaging with and experiencing the Nash equilibrium in real time, seeing the consequences of betrayal impact future decisions.”

Lee: “I was able to work with new members of the class and hear their perspectives... Each member had a different strategy and could explain their reasoning using math.”

Theme: Students thinking as PSTs

Students suggest improvements to the activity, or adaptations to their own (other) subject, as well as noting what they would do in their own classroom. This shows that they were thinking and evaluating the activity as learners but also as future teachers. In their study, Zarestky & Bigler (2021) too found that “PSTs were evaluating course activities as learners but also as professionals looking for strategies and activities to incorporate into their own future practice” (p. 61).

Lee: “In my class I would ensure that we scaffold the terms for each new scenario or stick to only two terms throughout to prevent students from feeling overwhelmed.”

Jess: “I was considering extensions to this problem, I found myself looking at it through an ELA [English Language Arts] teaching lens. I think that it would be interesting to use the Prisoner’s Dilemma to help students understand and work with characterization in an English class... For example, if they were doing a novel study on *The Hunger Games*, if a student was assigned Peeta’s character, then perhaps they would be more inclined to initially cooperate because he is often naive or trusting in the first novel of the series. Whereas Katniss’ character may or not cooperated [sic] based on who her partner is.”

Lucas: “[T]here would be merit in doing some rounds of this “game” blind, or unable to see who it is you are snitching on/undercutting... This could perhaps cut down on the amount of people who would ‘never under any circumstances snitch’ as a means of saving face, getting closer to the truer, colder equilibrium at the heart of the dilemma... Another modification may come through changing the amount of prison time, lessening the sentences to see how individuals might opt for a more beneficial outcome for the group, or increasing the sentences to exemplify the extremes of self-preservation.”

Discussion

The evidence from these four PSTs suggests that this simulation-based MSJ activity helped them to connect mathematics concepts from a game theory model, such as dominant strategy and Nash equilibrium, with real life situations. They describe changes in their conceptions of mathematics: seeing mathematics in ways they never had before, without a right-wrong dichotomy, and with the potential to be “a lot of fun”, “creative”, and “hands-on”. For these students, the mathematics in the activity was grounded in bodily and social experience, as something one “succumbs to”. Social interactions are described as contributing both to a reduction of anxiety and to deeper understanding of different perspectives around the dilemma of cooperation. The reflections also include connections between the activity and these students’ own experiences and interests. Connections made by students between the mathematics activity and their own subject matter and future classrooms indicate that they have gained knowledge or skills they are interested in transferring to their own practice. The analysis of the reflections suggests that, in these students’ experience, acting out a simulation involving interacting with peers around the classroom connected meaningfully with learning about a mathematics thought experiment and model.

Conclusion

Critical mathematics scholars “react against the tendency towards formalization of mathematics in schools that makes it irrelevant in students’ (and teachers’) lives” (Skovsmose & Greer, 2012, p. 379). The question of what is meaningful in mathematics education has a universal character, but arguably infinite possible personal responses. Although there is no one-size-fits-all answer for this question, simulation-based MSJ activities are a possible way to open the conversation with students within an educational context that does not dismiss the meaning-making potential of emotions, of social interactions, and of concern for real-world issues. This study cannot claim generality, as the data is from only four participants and one activity. Here, simulation-based MSJ activities are introduced and illustrated, and the data points towards their potential to provide cognitive and affective benefits to students’ understanding of, beliefs about, and connections with mathematics. Further studies will be required to deepen the investigation into these benefits.

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